Incomplete Property Rights and Overinvestment*

J. Atsu Amegashie

Department of Economics
University of Guelph
Guelph, Ontario
Canada N1G 2W1

E-mail: jamegash@uoguelph.ca
Phone: 519-824-4120 ext. 58945
Fax: 519-763-8497

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Abstract

I consider a model in which an asset owner must decide how much to invest in his asset mindful of the fact that an encroacher's valuation of the asset is increasing in the asset owner's investment. Due to incomplete property rights, the encroacher and asset owner engage in a contest over the control of the asset after investment has taken place. A standard result is that the asset owner will underinvest in the asset relative to the first-best level of investment when property rights are complete. To check the robustness of this result, I extend the benchmark model by changing (i) the nature of competition over property rights, (ii) the information that the players have about each other, (iii) the duration of the interaction between the players, and (iv) the bargaining power of the encroacher. Contrary to recent results, I find that when the interaction between the asset owner and the encroacher is infinitely repeated and the encroacher has some bargaining power over the size of the transfer from the asset owner to him, then there is a cooperative equilibrium in which the asset owner finds it optimal to over-invest in the asset when property rights are incomplete relative to the first-best level of investment when property rights are complete. Overinvestment is used to induce cooperation. However, this result depends on the nature of transfers or the encroacher's bargaining power.

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1. Introduction

It is widely accepted that the level of investment and creation of surplus in a country or by individuals in their assets depends on the security of property rights. Property rights affect economic performance. In recent years, this view has been forcefully expoused in De Soto (2000, 2001). De Soto (2001) argues that "[W]hat the poor lack is the easy access to the property mechanisms that could legally fix the economic potential of their assets, so that they could be used to produce, secure, or guarantee greater value in the expanded market … assets need a formal property system to produce significant surplus value."

To be sure, De Soto (2000, 2001) focuses on formal, legal, and *direct* protection of property rights of the kind provided by the state. However, since the protection of protection rights is costly, it is unlikely that the state can provide complete property rights. In a world of such incomplete property rights, individuals and private agents also invest in property rights protection. And the state cannot fully protect property rights through direct enforcement. Therefore, even if private agents or the state take actions to protect their property rights, these actions need not be only direct investments in fighting those who challenge their property rights. For example, Allen (2002), drawing on the insights of Demsetz (1967), shows that an asset owner might have the incentive to reduce the value of his asset in order to make the asset less attractive to encroachers. In particular, the asset owner might destroy attributes of the asset which are valued by the encroacher but not valued by owner or are valued less highly by him (i.e., the owner). Allen (2002) presents many interesting examples to illustrate his point. For example, he argues that Rhinoceros in Africa and elsewhere are dehorned to reduce their value to poachers. Also, he argues that Monsato, the American seed company, purchased the "terminator gene" to make its plants sterile (unable to germinate) in order to reduce the value of its seed to seed pirates. He also applies this simple idea to penal colonies, the quality of office furniture in public buildings, toilet paper and soap dispensers in public washrooms, the quality of children's skis, snowboards, bikes, and other interesting phenomena. Konrad (2002) also finds that incomplete property rights lead to underinvestment or cannot lead to overinvestment. He applies his analysis to investments by managers within firms and by autocrats within countries.[1] In a similar but more elaborate model, Gonzalez (2005) also finds that when property rights are incomplete, it may be optimal to adopt inferior technologies even if a superior technology is costless.[2] Finally, there is an empirical literature that shows that weaker protection of protection rights lead to lower investment (e.g., see Besley, 1995; Goldstein and Udry, 2008, and the references therein).

Clearly, in Allen (2002), Konrad (2002), Gonzalez (2005) and Goldstein and Udry (2008), a lower investment is used as a deterrent to encroachers and so overinvestment is not possible. For example, in Gonzalez (2005) there are two technologies: an existing but inferior technology and a superior technology which can be adopted at zero cost. If property rights are complete, the first-best solution will be the adoption of the superior technology. Since there are only two technologies, the second-best environment of incomplete property rights cannot lead to the adoption of a better

[1]More importantly, Konrad (2002) examines how the advantages of incumbency affect the investment incentives of the incumbent.
[2]In a growth model with incomplete property rights, Gonzalez (2007) and Gonzalez and Neary (2008) show that it may be optimal to reduce the rate of economic growth. This result is in the spirit of Allen (2002), Konrad (2002) and Gonzalez (2005) although the analysis is undertaken in a much richer dynamic environment.

technology than the one adopted in the first-best case. Therefore, overinvestment is not possible.[3]

While property rights affect investment in assets, investments in an asset can also affect property rights. Besley (1995) discusses this endogeneity issue in his econometric analysis. The use of investments to enhance property rights is evident in Razzaz's (1993) work on squatters in Jordan. Razzaz (1993, p. 351) notes that the settlers "… know that a makeshift shelter stands little chance and that the more they invest in permanent material the more their claim to the land is legitimized." This suggests that if the settlers had complete property rights over the land, they would have invested less which is an indication of over-investment when property rights incomplete.

The asset owner may increase the value of the asset for strategic reasons. A higher value of the asset could be seen as a commitment device by the owner to credibly communicate to the encroacher that he (i.e., the owner) is willingly to spend enough resources to protect it. This may cause the encroacher to reduce his effort. Even if the higher value causes the encroacher to increase his effort, the owner might still find it optimal to increase the value of the asset if the increase in his effort is sufficiently greater than the increase in the encroacher's effort such that his probability of keeping the asset or the proportion of it that he appropriates is sufficiently high and the increase in the cost of effort required to achieve this outcome is sufficiently low.

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[3]Gonzalez (2005) is, however, not primarily concerned with the comparison of investment levels in first-best and second-best environments. His focus is a different but interesting question: *given* a second-best world, if an agent can adopt an inferior technology and a superior but costless technology, will he necessarily adopt the superior technology?

Although the preceding argument is intuitive and may be a potential reason for overinvestment, I find that it is unable to generate overinvestment in both complete information and incomplete information environments. I therefore extend the complete-information case to an infinitely-repeated game setting and find that overinvestment may be undertaken because it facilitates cooperation between the asset owner and encroacher. In this case, the asset owner makes a transfer to the encroacher in each period and in return the encroacher promises not to challenge the property rights of the asset owner. However, I show that whether this results in overinvestment depends on bargaining power of the encroacher. The idea that transfers or redistribution can be used to induce cooperation, when property rights are incomplete, is not new. [4] The new result here is that the nature of transfers or the encroacher's bargaining power can lead to overinvestment.

My paper is also related to but different from a recent contribution by Robinson and Torvik (2005). In their paper, they opine that in the political economy of development, the issue of investments with negative social surplus is more important than underinvestment. In the case of Ghana under its first president, Kwame Nkrumah, Killick (1978, p.207) notes that "[T]he larger volume of 'investment' … could not compensate for the low-productivity uses to which it was put." Indeed, as Robinson and Torvik (2005, note 2) argue "[T]he problem under Nkrumah was not underinvestment … the consensus view is that the capital stock increased by 80% between 1960 and 1965 … The problem was in the way this investment was allocated." Using a model of political competition, Robinson and Torvik (2005) argue that such investments with negative social surplus (i.e., white elephants) could be seen as a credible promise to redistribute

[4]For example, see Amegashie (2008) and the references therein.

income to a segment of the electorate in order to influence the outcomes of elections in a world where politicians do not have complete property rights over power. However, my model and analysis differ from Robinson and Torvik (2005) in the following respects. First, I use a model of contest where efforts in the contest are not pure transfers. Second, overinvestment in my model does not lead to negative surplus. While overinvestment need not lead to a negative surplus, a negative surplus is an indication of overinvestment. My model cannot explain why an agent will invest in a white elephant while Robinson and Torvik (2005) cannot explain overinvestment that does not lead to a negative surplus. Indeed, while Robinson and Torvik (2005) present a very plausible theory to explain white elephants, their theory cannot explain the construction of projects like Ghana's recent multi-million dollar and controversial presidential palace[5] which was constructed in the capital, Accra, an ethnically diverse and metropolitan city in the Greater Accra region of the country. It is hard to believe that the NPP government's goal was to use the project to transfer resources to its political supporters given that its strongholds are in the Ashanti and Eastern regions of the country. Third, I show that even if investment is used as a transfer to induce cooperation, whether there is overinvestment depends on the nature of transfers. Finally, Robinson and Torvik (2005) do not undertake the analysis in section 4.

The seminal models of entry deterrence by Dixit (1980) and Spence (1977) also predict overinvestment. To the extent that an incumbent firm has to overinvest because it has incomplete property rights over market power, such models are related to the idea in this paper. A difference is that, unlike this paper, the investment undertaken by the

[5] See a report by BBC on Ghana's presidential palace at: http://news.bbc.co.uk/2/hi/africa/7720653.stm

incumbent firm does not have any positive value to the entrant. Therefore, investment is not used to make transfers to the entrant and so does not facilitate cooperation in the sense of this paper.

In addition to the literature on investment in the presence of incomplete property rights, my paper contributes to the new and small literature on signaling in contests. Horner and Sahuguet (2007), Munster (2009) and Zhang and Wang (2009) examine dynamic contests with signaling. However, in these papers, the efforts exerted in the contests in previous rounds or moves play the role of signals. This is not the case in my model. In my model, the owner's investment in the asset plays the role of a signal. Also, the valuations in my model are endogenous while they are exogenous in these papers.

The reminder of the paper is organized as follows: the next section considers a single-period two-stage model of investment and contest over property rights with complete information. Section 3 extends the model to the incomplete information case and section 4 considers an infinitely-repeated version of the model in section 2. Section 5 discusses the results and section 6 concludes the paper.

2. Investment and property rights

Consider a variant of the model of investment in the absence of complete property rights in Konrad (2002).[6] There are two risk-neutral agents, 1 and 2. Agent 1 owns an asset (e.g., a piece of land). Agent 2 is an encroacher who derives utility from using the asset. Let V = V(x) be the value of an asset to the owner when he invests x dollars in the asset. Assume that $x \geq 0$, V(0) = 0, $V'(x) > 0$ and $V''(x) < 0$. Let W = W(x) be the value of the asset to the encroacher, where W(0) = 0 and $W'(x) > 0$. Different valuations of the asset may be due to different abilities of the agents. For example, suppose when x is invested in the asset, it gives an intermediate input, n(x), which is used in a final production function $f_j(n(x))$, where $V(x) = f_1(n(x))$ and $W(x) = f_2(n(x))$, j = 1,2.

2.1 Complete property rights

Consider the benchmark case of complete property rights. In this case, the owner's first-best level of x is

$x^* = \arg\max V(x) - x$,

where x^* satisfies $V'(x^*) = 1$.

[6] Although the basic model in this section is similar to Konrad (2002), my results and focus in this paper differ from his. First, Konrad (2002) restricts his analysis to the case where the asset owner and the encroacher have the same valuation of the asset. He introduces asymmetry between the players by giving the owner a head-start advantage in his success probability; that is, the owner has some positive probability of success even if his effort is zero and the thief exerts a positive effort. Konrad (2002) is primarily interested in this head-start advantage as the title of his paper indicates. Second, and most importantly, the analyses in sections 3 and 4 of this paper are different from Konrad (2002).

2.2 Incomplete property rights with complete information

Now consider the case of incomplete property rights. Suppose the owner invests e_1 dollars in protecting his property and the encroacher invests e_2 dollars to challenge the property rights of the owner.

Assume that the probability that the owner will successfully protect his asset is $p_1 = \frac{e_1}{e_1 + e_2}$. Accordingly, the probability that the encroacher will be successful is $p_2 = 1 - p_1$. This is an all-or-nothing, winner-takes-all contest. However, we can also interpret these probabilities as the proportions of the asset that each person can control or use.

The probability function above is known as the contest success function and the particular form used here is referred to as an imperfectly-discriminating contest success function because the party with the higher effort does not win with certainty. When the party with the higher effort wins with certainty, the contest is perfectly discriminating and is referred to as an all-pay auction (e.g., see Konrad, 2009). I return to this distinction in section 5.

I model the game as a two-stage game. In the first stage the owner chooses x and in the second stage, the encroacher and owner choose e_1 and e_2 simultaneously in a complete-information contest. I look for a subgame perfect equilibrium by backward induction.

Working backwards, consider stage 2. In this stage, noting that x is sunk, the players' payoffs are

$$U_1 = p_1 V(x) - e_1 \tag{1}$$

and

$$U_2 = p_2 W(x) - e_2 \tag{2}$$

The unique pure-strategy Nash equilibrium values, after some algebra, are $\hat{e}_1 = \frac{V^2 W}{(V+W)^2}$ and $\hat{e}_2 = \frac{W^2 V}{(V+W)^2}$ (see, for example, Nti, 1999; Konrad, 2009). The equilibrium probability for the owner is $\hat{p}_1 = \frac{V}{V+W}$. The owner's payoff is $U_1^N = \frac{V^3}{(V+W)^2} > 0$ and the encroacher's payoff is $U_2^N = \frac{W^3}{(V+W)^2} > 0$. Note that for any x, $0 < \hat{p}_1 < 1$. I suppress the dependence of W and V on x for notational convenience whenever necessary.

In stage 1, the owner chooses x to maximize

$$S_1(x) = \frac{V^3}{(V+W)^2} - x, \tag{3}$$

Taking the derivative of (3) with respect to x and evaluating at x* noting that $V'(x^*) = 1$ and simplifying gives

$$\left.\frac{\partial S_1}{\partial x}\right|_{x=x^*} = \frac{-2V^3 W'(x^*) - 3VW^2 - W^3}{(V+W)^3} < 0 \tag{4}$$

Let $\hat{x}$ = arg max $S_1(x)$. This is the asset-owner's optimal level of investment when property rights are incomplete.[7] Then (4) implies that $\hat{x} < x^*$. Hence there is underinvestment when property rights are incomplete.

[7] I assume that the second-order condition for a local maximum holds. If $W(x) = \beta V(x)$, then $S_1(x) = V(x)/(1 + \beta) - x$, where β is a positive parameter. Clearly, in this case, the optimal investment is an interior solution and is a unique global maximum.

The underinvestment result holds if the contest is an all-pay auction. The proof is straightforward. Suppose $V(x) > W(x)$ for all x. Then, as shown by Hillman and Riley (1989) and Baye et al (1996), the owner's expected payoff in a mixed-strategy equilibrium in the contest is $V(x) - W(x)$. Therefore, he will choose x to maximize $V(x) - W(x) - x$. Then given $W'(x) > 0$, it is easy to show that the owner will choose $x < x^*$. If $V(x) \leq W(x)$ for all x, then the owner's equilibrium payoff in the contest is zero. Therefore, his optimal investment is zero.[8]

The analysis gives the following proposition:

Proposition 1: *In the subgame perfect equilibrium of the finite-period investment-cum-contest game the owner's level of investment in the asset when property rights are incomplete will be smaller than his level of investment when property rights are complete.*

Proposition 1 is robust to changing the timing of moves in the contest or the use of the generalized Tullock contest success function (e.g., see Konrad (2009) for a discussion of this function). It is consistent with the results of Allen (2002), Konrad (2002), Gonzalez (2005) and the other papers cited in section 1.[9]

3. Incomplete property rights with incomplete information

Given that the basic model in the previous section cannot produce overinvestment, I extend it by introducing incomplete information in the contest to see if

[8] Note that even if $V(x) > W(x)$ for all or $V(x) \leq W(x)$ for all x does not hold, the underinvestment result will still hold because in stage 1, the asset owner knows that any x chosen will give a payoff of either zero or $V(x) – W(x)$ in stage 2.

[9] Proposition 1 also holds if investment and effort decisions are made simultaneously. To make a different point, suppose $V(0) = W(0) > 0$. This may be the case because the asset (e.g., a piece of land for grazing) may have value even if it is not maintained. Then if W(x) rises sufficiently faster than V(x), we can show that the asset owner will choose x = 0 even if investment is costless. That is, $S_1(0) > S_1(x) + x$ for $x > 0$.

a signaling motive can explain over-investment in the asset when property rights are incomplete. While incomplete property rights give rise to a contest over property rights, it does not necessarily give rise to incomplete information. Yet, it is conceivable that when there is a contest, the contestants may have incomplete information about each other.

Suppose that an asset-owner can have two valuations: a high valuation, $V_H(x)$, and low-valuation, $V_L(x)$, where $V_H(x) > V_L(x)$ and $V_j(x)$ has the same properties as before, j = H, L. The probability that the asset-owner has a high valuation is q and the probability that he has a low valuation is $1 - q$, where $0 < q < 1$. Suppose that the asset-owner's valuation is his private information but the encroacher knows the distribution of these valuations. The encroacher has valuation $W(x_j)$ which is common knowledge, j = H, L. Since the contest is not repeated, there is no way of signaling one's type through the effort expended in previous rounds, and hence no further insight is gained over and beyond the no-signaling papers of Hurley and Shogren (1998a, 1998b) by making the encroacher's valuation his private information.

An asset-owner will use his investment in the asset to signal his type to the encroacher. I look for a perfect Bayesian equilibrium of the game.

Let x_j^* be the investment level of an asset owner of type V_j when property rights are complete, j = H, L. Therefore, $V_j'(x_j^*) = 1$ and $x_H^* > x_L^* > 0$. Also, let $\hat{x}_H$ and $\hat{x}_L$ be the investment levels of the high-valuation and low-valuation types if property rights are incomplete but there is complete information. It is easy to show that $\hat{x}_H > \hat{x}_L > 0$.

3.1 Pooling equilibrium

A pooling equilibrium potentially offers the hope of finding an equilibrium with overinvestment. Therefore, consider a pooling equilibrium. If the equilibrium is pooling, then conditional on x^p, the contest in stage 2 is a similar to the no-signaling models of Hurley and Shogren (1998a, 1998b).[10]

Proposition 2: *There is no pooling equilibrium where both types of the asset owner invest* $x_L = x_H = x^p > 0$.[11]

The proof is as follows: For my purposes and without loss of generality, I consider a pooling equilibrium in which both types invest $x^p > x_L^* > 0$ (i.e., at least, one type overinvests). For this to be an equilibrium we have to show that no type of the asset owner has a profitable deviation.

It is convenient to define τ as the encroacher's belief that the asset owner has a high valuation. The encroacher chooses his effort, e_2^p to maximize

$$U_2^p = \left(\tau \frac{e_2^p}{e_2^p + e_{1H}^p(V_H)} + (1-\tau) \frac{e_2^p}{e_2^p + e_{1L}^p(V_L)} \right) W - e_2^p, \tag{5}$$

where I have suppressed the dependence of the valuations on x^p. It is easy to show that the best response function of a type j asset owner (i.e., the informed player) is $e_{1j}^p(V_j) = \sqrt{e_2^p V_j} - e_2^p$, j = H, L. Then taking the derivative of (5) with respect to e_2^p, substituting the best-response functions of the high-valuation and low-valuation types of the asset owner and solving gives

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[10] Hurley and Shogren (1998a) consider continuous valuations while they consider discrete valuations in Hurley and Shogren (1998b).

[11] Note that $x^p = 0$ cannot be an equilibrium. If this were an equilibrium, there will be no contest. But if there is no contest, then $x^p = 0$ is not an asset owner's best response.

$$\hat{e}_2^p(\tau) = \left(\frac{\tau V_L \sqrt{V_H} + (1-\tau) V_H \sqrt{V_L}}{W(\tau V_L + (1-\tau) V_H) + V_L V_H} \right)^2 W^2, \tag{6}$$

where $\tau = q$ in a pooling equilibrium. It is easy to verify that equation (6) boils down to the equilibrium effort in the complete-information case (i.e., $V_L = V_H = V$).

We can write the equilibrium payoff of an asset owner of type j in stage 1 as

$$S_j(x^p) = \frac{\hat{e}_{1j}^p}{\hat{e}_{1j}^p + \hat{e}_2^p} V_j(x^p) - \hat{e}_{1j}^p - x^p, \tag{7}$$

where $\hat{e}_{1j}^p(V_j) = \sqrt{\hat{e}_2^p V_j} - \hat{e}_2^p$ and j = H, L.

In a pooling equilibrium, we require both types of the asset owner to exert a positive effort. This condition is satisfied if the low-valuation type of the asset owner exerts a positive effort even if the encroacher believes with certainty that he is a high-valuation type (i.e., $\tau = 1$). It can be shown that given $\tau = 1$, $\hat{e}_{1L}^p(V_L) = \sqrt{\hat{e}_2^p V_L} - \hat{e}_2^p > 0$ if $(W + V_H)\sqrt{V_L} - W\sqrt{V_H} > 0$.

Putting the best response function $e_{1j}^p(V_j) = \sqrt{e_2^p V_j} - e_2^p$ into (7), we can rewrite the equilibrium payoff as

$$S_j(x^p) = [V_j(x^p) - x^p] - 2\sqrt{\hat{e}_2^p V_j(x^p)} + \hat{e}_2^p \tag{8}$$

Note that $\hat{e}_{1j}^p(V_j) = \sqrt{\hat{e}_2^p V_j(x^p)} - \hat{e}_2^p > 0$ implies that $\sqrt{V_j(x^p)/\hat{e}_2^p} > 1$. Therefore, the sum of the last two terms in (8) is negative. This inequality also implies that the sum of the last two terms in (8) is decreasing in $\hat{e}_2^p$. That is, the absolute magnitude of the sum of the last two terms is increasing in $\hat{e}_2^p$. Therefore, all things being equal, an asset owner is worse off when $\hat{e}_2^p$ increases.

We know that $0 < \tau = q < 1$ in a pooling equilibrium. I impose the following out-of-equilibrium beliefs. When the encroacher observes an investment greater (less) than x^p, his belief that the asset owner has a high (low) valuation is, greater (less) than his equilibrium belief. In particular, I assume that (a) $\mathrm{pr}\left(V_H \middle| x > x^p\right) = \mu_H$ is sufficiently close to 1 $\Rightarrow \mathrm{pr}\left(V_L \middle| x > x^p\right) = 1 - \mu_H$ is sufficiently close to zero, and (b) $\mathrm{pr}\left(V_L \middle| x < x^p\right) = \mu_L$ is sufficiently close to 1 $\Rightarrow \mathrm{pr}\left(V_H \middle| x < x^p\right) = 1 - \mu_L$ is sufficiently close to zero.

Next, we note that

$$\mathrm{sign}\frac{\partial \hat{e}_2^p}{\partial \tau} = \mathrm{sign}\left(W - \sqrt{V_L V_H}\right) \tag{9}$$

Therefore, $\partial \hat{e}_2^p / \partial \tau$ has an ambiguous sign. That is, the effect of the encroacher's belief that the asset owner has a high valuation on the encroacher's equilibrium effort is ambiguous.

Note that a deviation from a pooling equilibrium will have two effects on the encroacher's effort through its effect on (i) the valuations of the players, and (ii) the encroacher's belief that the asset owner has a high valuation. Formally,

$$d\hat{e}_2^p = \frac{\partial \hat{e}_2^p}{\partial V_L} dV_L + \frac{\partial \hat{e}_2^p}{\partial V_H} dV_H + \frac{\partial \hat{e}_2^p}{\partial W} dW + \frac{\partial \hat{e}_2^p}{\partial \tau} d\tau . \tag{9a}$$

The first three terms on the RHS of (9a) represent the cumulative effect of a change in x on the encroacher's effort holding τ fixed while the fourth term is the effect of a change in τ on the encroacher's effort holding the players' valuations fixed. Consider the following three exhaustive cases:

Case (i): Suppose $\partial \hat{e}_2^p / \partial \tau > 0$. Suppose the asset owner deviates to an investment level, $x^p - \varepsilon > 0$ where ε is *very small* but positive. That is, the asset owner deviates to an investment level *marginally* lower than x^p. This deviation has only an *infinitestimally* small effect on the valuations of the players and the asset owner loses an *infinitestimally* small surplus from investment relative to the surplus (i.e., the term in square brackets in (8)) generated at x^p.[12] However, he gains in the contest because the encroacher's belief, $1 - \mu_L$, that the asset owner is a high-valuation type is sufficiently lower than his belief, q, in the pooling equilibrium leading to a reduction in the encroacher's effort [i.e., $\hat{e}_2^p(\tau = 1 - \mu_L) < \hat{e}_2^p(\tau = q)$]. Formally, put $dV_j \equiv V_j(x^p - \varepsilon) - V_j(x^p) \approx 0$, $dW \equiv W(x^p - \varepsilon) - W(x^p) \approx 0$, $d\tau \approx (1 - \mu_L) - q < 0$, and $\partial \hat{e}_2^p / \partial \tau > 0$ into equation (9a) to obtain $d\hat{e}_2^p < 0$, j = H, L. Hence, the asset owner will deviate from a pooling equilibrium.

Case (ii): Suppose $\partial \hat{e}_2^p / \partial \tau < 0$. Suppose the asset owner deviates to an investment level, $x^p + \varepsilon > 0$ where ε is very small but positive. That is, the asset owner deviates to an investment level *marginally* higher than x^p. By doing so, he may lose an *infinitestimally* small surplus from investment relative to the surplus (i.e., the term in square brackets in (8)) generated at x^p. However, he gains in the contest because the encroacher's belief, μ_H, that he is a high-valuation type is sufficiently higher than his belief, q, in the pooling

[12]He may even gain by reducing his investment if x^p is greater than his first-best level of investment.

equilibrium which gives $\hat{e}_2^p(\tau = \mu_H) < \hat{e}_2^p(\tau = q)$. Again, using equation (9a), noting that $dV_j = dW \approx 0$, $d\tau \approx \mu_H - q > 0$, and $\partial\hat{e}_2^p / \partial\tau < 0$ gives $d\hat{e}_2^p < 0$, j = H, L. Hence, the asset owner will deviate from a pooling equilibrium.

Case (iii): Suppose $\partial\hat{e}_2^p / \partial\tau = 0$. Then the fourth term on the RHS of (9a) is zero. Note that very small deviations from x^p will have a very small effect on an asset-owner's payoff relative to his payoff at x^p. But under a monotonicity condition (see below), there is a profitable and discrete deviation from the pooling equilibrium for, at least, one type of the asset owner. This exhausts all the possible cases and so completes the proof that there is no pooling equilibrium with overinvestment given the specific out-of-equilibrium beliefs.

There is also no partial pooling equilibrium in stage 1 where both types of the asset owner invest $x_L = x_H = x^p > x_L^*$ with probability $\rho \in (0,1)$ and invest $x_j^p \neq x^p$ with the remaining probability, 1 – ρ, j = H, L. Towards a contradiction, suppose that this equilibrium exists. Then for such a non-degenerate mixed-strategy equilibrium, we require that an asset owner must be indifferent between investing $x_L = x_H = x^p$ and investing $x_j^p \neq x^p$, j = H, L. For a partial pooling equilibrium, we also require $x_L^p \neq x_H^p$. Then when the encroacher observes x_L^p or x_H^p, he will correctly infer the asset-owner's type. Hence, x_L^p and x_H^p must respectively be equal to the asset-owner's optimal investments in the full-information equilibrium. Therefore, $x_H^p = \hat{x}_H > x_L^p = \hat{x}_L$. Using the same out-of-equilibrium beliefs as before *and* noting that $\mathrm{pr}(V_H | x = \hat{x}_H) = \mathrm{pr}(V_L | x = \hat{x}_L) = 1$, it is easy to see that the same arguments for a profitable deviation as in the case of $\rho = 1$ are applicable whereby, at least, one type of the asset owner will deviate from x^p. Therefore, there is no partial pooling equilibrium.

It is well known that because the solution concept of perfect Bayesian equilibrium places almost no restrictions on out-of-equilibrium beliefs, it is able to support several equilibria. Therefore, one may argue that proposition 2 is not general enough because it was based on a specific set of out-of-equilibrium beliefs. The issue then boils down to the use of refinements in the choice of these beliefs (i.e., reasonable beliefs). Indeed, the out-of-equilibrium beliefs above were such that there were jumps in the encroacher's beliefs for very small deviations from $x^p > x_L^* > 0$. Such small deviations from x^p kept the valuations almost unchanged allowing us to ignore changes in the contest due to changes in valuations. The alternative would be to assume that deviations from x^p within a certain interval have almost no effect on the encroacher beliefs. That is, $\mathrm{pr}\left(V_H \middle| x^p - \underline{\varepsilon} \le x \le x^p + \overline{\varepsilon}\right) \approx q$, where $x^p - \underline{\varepsilon} > 0$, and $\underline{\varepsilon}$, $\overline{\varepsilon} > 0$. This out-of-equilibrium belief implies that, starting from a pooling equilibrium where $\tau = q$, $d\tau \approx 0$ for $x \in [x^p - \underline{\varepsilon}, x^p + \overline{\varepsilon}]$. Putting $d\tau \approx 0$ into (9a) leaves only the first three terms on the RHS of (9a) similar to case (iii) above. If $\hat{e}_2^p$ is monotonic in x on the interval $[x^p - \underline{\varepsilon}, x^p + \overline{\varepsilon}]$ holding τ fixed,[13] then there exists an investment level in this interval such that the encroacher reduces his effort. Then, at least, one type of the asset owner will deviate. Therefore, under reasonable assumptions, proposition 2 continues to hold given this

[13] This holds if W(x) increases at a sufficiently fast rate relative to the rate for $V_L(x)$ and $V_H(x)$. For example, suppose $W = x^\alpha$, $V_L = x^\beta$, $V_H = x^\gamma$, where $0 < \gamma, \beta, \alpha < 1$ and these parameters are growth rates. Then the encroacher's effort is increasing in x if α is sufficiently high. For example, if $\alpha = 0.3$, $\beta = 0.4$, and $\gamma = 0.5$, then the encroacher's effort is increasing in x for $\tau \in [0,1]$ and any $\alpha \geq 0.3$ will also yield this result. Of course, the encroacher's effort could be decreasing in x even if W(x) does not rise fast enough. This would also satisfy the monotonicity condition.

alternative set of out-of-equilibrium beliefs. Indeed, holding τ fixed, I could not find an example where the encroacher's effort was not monotonic in the neighborhood of x_L^* and x_H^*.[14] To be sure, one could assume that the encroacher's beliefs are such that any deviation from x^p increases his belief that the asset owner has a high valuation and leads to an increase in his effort, *regardless* of whether that deviation led to an increase or decrease in the asset-owner's investment. Under this assumption, one could find a pooling equilibrium with overinvestment. However, such an assumption is questionable. Consequently, I abandon the quest to find a pooling equilibrium with overinvestment.[15]

3.2 Separating equilibrium

Having abandoned the quest to find a pooling equilibrium with overinvestment, I now turn to finding a separating equilibrium for the sake of completeness and more importantly to show that underinvestment may not be driven by the incompleteness of information.

Recall that $\hat{x}_H$ and $\hat{x}_L$ are the investment levels of the high-valuation and low-valuation types if property rights are incomplete but there is complete information.

[14] Using the functions in footnote 13 subject to the restriction that $\gamma > \beta$ and holding τ fixed, I find that, for several parameter values, the encroacher's effort is increasing in x on the domain $[0, 5x_H^*]$. I could not find an example where this was not the case.

[15] It is also interesting to note that Munster (2009) obtains a proposition similar to proposition 2 where in his two-stage repeated contest with efforts in stage1 as signals, there is no equilibrium in stage 1 where the low-valuation and the high-valuation types choose the same effort with probability one. However, in his case, he finds that a partial pooling equilibrium exists. In Munster's (2009) model of two-sided asymmetric information, a particular feature makes updated beliefs about types easier. The low-valuation type has a valuation of zero, so any positive bid is strictly dominated for this type. This means that if a player sees that his opponent's bid in stage 1 is positive, his belief is that his opponent is a high-valuation type.

Proposition 3: *It is possible to construct a perfect Bayesian separating equilibrium with underinvestment where the low-valuation type invests* $\hat{x}_L < x_L^*$ *and the high-valuation type invests* $\hat{x}_H < x_H^*$*, where* $\hat{x}_H > \hat{x}_L$ *with equilibrium posterior beliefs* $\mathrm{pr}\left(V_H \middle| x = \hat{x}_H\right) = 1$ *and* $\mathrm{pr}\left(V_L \middle| x = \hat{x}_L\right) = 1$. *The out-of-equilibrium beliefs are* $\mathrm{pr}\left(V_H \middle| \hat{x}_L < x < \hat{x}_H \cup x > \hat{x}_H\right) = 1$ *and* $\mathrm{pr}\left(V_L \middle| x < \hat{x}_L\right) = 1$.

To demonstrate this proposition, we proceed as follows: In a separating equilibrium, there is complete information since a player's type is fully revealed. Therefore, the payoff of the high-valuation type is

$$S_H(\hat{x}_H) = U_H(\hat{x}_H) - \hat{x}_H \tag{10}$$

where $U_H(\hat{x}_H) = \dfrac{(V_H(\hat{x}_H))^2 W(\hat{x}_H)}{(V_H(\hat{x}) + W(\hat{x}_H))^2}$ is his equilibrium payoff in the contest.

Similarly, the low-valuation type's payoff is

$$S_L(\hat{x}_L) = U_L(\hat{x}_L) - \hat{x}_L \tag{11}$$

where $U_L(\hat{x}_L) = \dfrac{V_L(\hat{x}_L)(W(\hat{x}_L))^2}{(V_L(\hat{x}_L) + W(\hat{x}_L))^2}$ is his equilibrium payoff in the contest.

We have to show that no player has a profitable deviation. First, note that no type will deviate from his equilibrium investment if such a deviation will not misrepresent his type. Given the out-of-equilibrium beliefs in proposition 3, the high-valuation type may misrepresent his type by deviating to $x_H^d \le \hat{x}_L$. In this case, the encroacher will think that he is a low-valuation type. Let $e_{IH}^d(V_H)$ be the effort of the high-valuation type when he deviates. However, believing that his opponent has low valuation, the encroacher chooses his effort, e^d_{2L}, in the contest to maximize

$$\Pi^d_{2L} = \frac{e^d_{2L}}{e^d_{2L} + e^d_{1H}(V_L)} W(x^d_H) - e^d_{2L} \tag{12}$$

Note that the true best response function of an asset-owner of type V_j is

$$e^d_{1j}(V_j) = \sqrt{e^d_{2L} V_j} - e^d_{2L} \tag{13}$$

j =H, L. However, when the high-valuation type deviates, the encroacher believes that his best-response function is $e^d_{1H}(V_L) = \sqrt{e^d_{2L} V_L} - e^d_{2L}$. Putting this into $\partial\Pi^d_{2L} / \partial e^d_{2L} = 0$ and solving gives[16]

$$\hat{e}^d_{2L} = V_L W^2 / (W + V_L)^2 . \tag{14}$$

The high-valuation type puts $e^d_{2L} = V_L W^2 / (W + V_L)^2$ into his true best response function $e^d_{1H}(V_H) = \sqrt{e^d_{2L} V_H} - e^d_{2L}$ to get

$$\hat{e}^d_{1H} = \frac{W(V_L + W)\sqrt{V_H V_L} - V_L W^2}{(V_L + W)^2} \tag{15}$$

In stage 1, the high-valuation type chooses x^d_H to maximize

$$S^d_H(x^d_H) = U^d_H(x^d_H) - x^d_H \tag{16}$$

where $U^d_H(x^d_H) = \frac{\hat{e}^d_{1H}}{\hat{e}^d_{1H} + \hat{e}^d_{2L}} V_H - \hat{e}^d_{1H}$ and the valuations in (11) are functions of x^d_H.

Let $\hat{x}^d_H$ be his optimal deviation. The corresponding payoff is $S^d_H(\hat{x}^d_H)$.

[16] One can also obtain the expression in (14) by putting $\tau = 0$ into equation (6).

Now consider the low-valuation type. Given the out-of-equilibrium beliefs in proposition 3, he may deviate to some $x_L^d > \hat{x}_L$ in order to signal that he is a high type. As before, he will choose

$$e_{1L}^d = \frac{W(V_H + W)\sqrt{V_H V_L} - V_H W^2}{(V_H + W)^2}, \tag{17}$$

and the encroacher, believing that his opponent is a high-valuation type, will choose

$$\hat{e}_{2H}^d = V_H W^2 / (W + V_H)^2 \tag{18}$$

in the contest.

In stage 1, the equilibrium payoff of the low-valuation asset owner is

$$S_L^d(\hat{x}_L^d) = U_L^d(\hat{x}_L^d) - \hat{x}_L^d \tag{19}$$

where $U_L^d(\hat{x}_L^d) = \dfrac{\hat{e}_{1L}^d}{\hat{e}_{1L}^d + \hat{e}_{2H}^d} V_L - \hat{e}_{1L}^d$ and the valuations are functions of $\hat{x}_L^d$.

The separating equilibrium in proposition 3 exists if

$$S_H(\hat{x}_H) \geq S_H^d(\hat{x}_H^d) \tag{20}$$

and

$$S_L(\hat{x}_L) \geq S_L^d(\hat{x}_L^d) \tag{21}$$

Note that, in equilibrium, beliefs are updated using Bayes rule. For example,

$$pr(V_H | x = \hat{x}_H) = \frac{pr(\hat{x}_H | V_H) pr(V_H)}{pr(\hat{x}_H | V_H) pr(V_H) + pr(\hat{x}_H | V_L) pr(V_L)} = \frac{1 \times q}{1 \times q + 0 \times (1 - q)} = 1 .$$

Next, we shall try to glean further insight into why a separating equilibrium is possible to construct. In a separating equilibrium, the effort of the encroacher when he faces a high-valuation type is $\hat{e}_{2H}^s = V_H(\hat{x}_H)[W(\hat{x}_H)]^2 / (W(\hat{x}_H) + V_H(\hat{x}_H))^2$. Using the high-valuation type's best response function in a separating equilibrium gives $\hat{e}^s_{1H}(V_H) = \sqrt{\hat{e}^s_{2H} V_H} - \hat{e}^s_{2H}$. Putting this into $S_H(\hat{x}_H) = V_H \hat{e}^s_{1H} / [\hat{e}^s_{1H} + \hat{e}^s_{2H}] - \hat{e}^s_{1H} - \hat{x}_H$ and simplifying, we can rewrite his payoff function in a separating equilibrium as

$$S_H(\hat{x}_H) = [V_H(\hat{x}_H) - \hat{x}_H] - 2\sqrt{\hat{e}^s_{2H} V_H(\hat{x}_H)} + \hat{e}^s_{2H} \tag{22}$$

If the high-valuation type misrepresents his type, the encroacher's effort is $\hat{e}^d_{2L} = V_L(\hat{x}^d_H)[W(\hat{x}^d_H)]^2 / (W(\hat{x}^d_H) + V_L(\hat{x}^d_H))^2$. Similarly, we can substitute $\hat{e}^d_{1H}(V_H) = \sqrt{\hat{e}^d_{2L} V_H} - \hat{e}^d_{2L}$ into (16) to get

$$S_H(\hat{x}^d_H) = [V_H(\hat{x}^d_H) - \hat{x}^d_H] - 2\sqrt{\hat{e}^d_{2L} V_H(\hat{x}^d_H)} + \hat{e}^d_{2L} \tag{23}$$

As before, $\hat{e}^s_{1H}(V_H) = \sqrt{\hat{e}^s_{2H} V_H(\hat{x}_H)} - \hat{e}^s_{2H} > 0$ implies that $\sqrt{V_H(\hat{x}_H) / \hat{e}^s_{2H}} > 1$. Therefore, the sum of the last two terms in (22) is negative. The same is true of the last two terms in (23) because $\sqrt{V_H(\hat{x}^d_H) / \hat{e}^d_{2L}} > 1$. Also, these inequalities imply that the sum of the last two terms in (22) is decreasing in $\hat{e}^s_{2H}$ while the sum of the last two terms in (23) is decreasing in $\hat{e}^d_{2L}$.

Suppose, for a moment, that $\hat{x}_H = \hat{x}^d_H$. Then $\hat{e}^s_{2H} > \hat{e}^d_H$. Therefore, all things being equal, if the high-valuation type misrepresents his type, the encroacher reduces his effort relative to his effort in the separating equilibrium. Then given that the sum of the last two terms in (22) is negative and that this term is decreasing in the effort of the encroacher, it follows that the high-valuation asset owner is better off by misrepresenting his type.[17] However, given that $x_H^* = \arg\max\,[V_H(x_H) - x_H] > \hat{x}_H > \hat{x}_L \geq \hat{x}_H^d$, it follows that the size of the first-term in square brackets in (22) is bigger than it is in (23). Hence, this effect makes the asset-owner worse off if he deviates. If this latter effect dominates the former effect, then the high-type asset owner will not deviate. A reverse but similar argument with two opposing effects also explains why the low-valuation type will not deviate from a separating equilibrium. In this case, the benefit of deviating from $\hat{x}_L$ and moving to a higher level of investment which is closer to x_L^* must be outweighed by the cost of the encroacher believing that the low-valuation type is stronger than he actually is.[18] Note that these conditions are independent of q and $1-q$ (i.e., the prior beliefs) because the out-of-equilibrium beliefs are independent of the prior beliefs. However, regardless of the out-of-equilibrium beliefs chosen, the important point to note is that there cannot be overinvestment in a separating equilibrium because there is complete information in this equilibrium and therefore, *for a given type*, proposition 1 holds.

In the preceding paragraph, the size of each of the opposing effects is increasing in the difference between V_H and V_L. This explains why, in proposition 3, I do not make the apparently intuitive claim that a separating equilibrium exists if the difference between V_H and V_L is sufficiently high.

Finally, it remains to show that there is, indeed, a specific example where the inequalities in (20) and (21) hold. As an example, suppose $V_H(x_H) = 2(x_H)^{0.5}$,

[17]This is the benefit of being underestimated in the contest which is also the case in Munster (2009) where the contestants have exogenous valuations. However, because valuations are endogenous in this case, there is a countervailing welfare loss from moving further away from the first-best level of investment. This will be obvious shortly. In Munster (2009), the countervailing effect is a lower probability of success due to a reduction in effort in the first round.
[18]All things being equal, the encroacher's effort is higher if he believes that asset owner is a high-valuation type.

$V_L(x_L) = (x_L)^{0.5}$, and $W(x_j) = (x_j)^{0.5}$, j = H, L. Then $x_L^* = 0.25$, $\hat{x}_L = 0.0156$, $x_H^* = 1$ and $\hat{x}_H = 0.1975$. In addition, $S_H(\hat{x}_H) = 0.1975$ and $S_L(\hat{x}_L) = 0.0156$.

Given the specific functional forms, $\hat{x}_H^d = 0.324 > \hat{x}_L = 0.0156$ which is not consistent with $x_H^d \leq \hat{x}_L$. In fact, any $x_H^d \leq \hat{x}_L$ gives $S_H^d(x_H^d) < 0$.[19] Therefore, the high-valuation type will not deviate from a separating equilibrium. Also, using equations (17), (18) and (19) and the specific functional forms, we find that any $x_L^d > \hat{x}_L = 0.0156$ gives $S_L^d(x_L^d) < 0$. Hence the low-valuation type will also not deviate. Therefore, given the specific functions above, it is possible to construct a perfect Bayesian separating equilibrium with underinvestment.

4. Infinitely-repeated interaction and incomplete property rights

Consider an infinitely-repeated version of the model in section 2. Without loss of generality, suppose W(x) = V(x). In *each* period, the asset owner makes an investment choice and then possibly engages in a contest with the encroacher. I shall show that there can be overinvestment in this environment.

Suppose the encroacher and the asset owner decide to negotiate a self-enforcing peaceful agreement. In each period, the asset owner will give the encroacher an upfront transfer. In return, the encroacher will not challenge the property rights of the asset owner. Since in the benchmark game in section 2, the asset owner chooses his investment before the contest over property rights, I maintain consistency in the timing of moves by

[19] This part of the analysis was undertaken with the help of the math software, Maple. I also looked at plots of the objective functions over the relevant domains.

assuming that the asset owner chooses his investment before the parties bargain over the size of the transfer.

Suppose that the asset owner can commit to an agreement but the encroacher cannot.[20] The asset owner uses a Nash reversion strategy (trigger strategy) where he punishes the encroacher by reverting to the Nash equilibrium play forever if the encroacher reneges on the agreement.[21] Let $\delta \in [0,1)$ be the encroacher's discount factor.

The timing of actions is as follows. In each period:

(i) The asset owner chooses the level of investment, x,

(ii) The asset owner and the encroacher bargain over the size of the transfer from the asset owner to the encroacher.

(iii) If they agree on the transfer and the encroacher does not renege, the game ends and the sequence of actions is repeated in the next period.[22] If the encroacher reneges, the non-cooperative game is played forever in subsequent periods.

(iv) If there is no agreement in (iii), there is a contest over control of the asset. Thereafter, we are back to (i) and the sequence of actions is repeated in the next period.

[20]This assumption does not affect the analysis. Besides commitment by the asset owner is a reasonable assumption since he has to honor his side of the agreement first (i.e., make a transfer to the encroacher) before the encroacher honors his side of the agreement. In other words, the asset owner cannot betray the encroacher in any period.

[21]For a recent and interesting analysis using a Nash reversion strategy, see Conconi and Sahuguet (2009). It is well known that the strategies in Abreu (1986, 1988) can sustain cooperation in cases where a Nash reversion strategy fails to do so. However, using a Nash reversion strategy is sufficient to prove that there could be overinvestment. More effective strategies will not change this result.

[22] Of course, after several periods of successful bargaining agreements, a norm will develop under which there is no further need to bargain and the parties simply use the transfer rule used in previous periods. However, since bargaining is costless in my model, it really does not matter whether they bargain in every period.

4.1 Equilibrium analysis

Since the game is stationary in each period, the non-cooperative subgame perfect Nash equilibrium in the contest is the equilibrium of static version of the game in section 2. Therefore, in the contest, the non-cooperative subgame perfect Nash equilibrium gives the asset owner $U_1^N(\hat{x}) = 0.25V(\hat{x})$ and the encroacher $U_2^N(\hat{x}) = 0.25V(\hat{x})$ in each period.

Let the transfer from the asset owner to the encroacher be $S_2^C = \Omega$. Then the asset owner gets $S_1^C = V(x) - \Omega - x$. For each party to participate in a cooperative arrangement we require the following necessary conditions:

$$S_2^C \geq U_2^N(\hat{x}) \equiv S_2^N \tag{24}$$

and

$$S_1^C \geq S_1^N \equiv U_1^N(\hat{x}) - \hat{x}\,. \tag{25}$$

If the encroacher reneges on the agreement, he will expend a positive but small effort, ε, in the contest and, given the contests success function, appropriate the entire asset with certainty (i.e., $p_2 = 1$ if $e_2 > 0$ and $e_1 = 0$). So the encroacher's payoff, if he deviates, is $S_2^D = V(x) - \varepsilon + \Omega$.[23] Using well-known arguments, it can easily be shown that given that the asset owner uses a Nash reversion strategy, the encroacher will not deviate in any period if

[23] I assume that the encroacher does not invest in the asset because he does not have the power to take investment decisions. This is consistent with the subsequent example of political patronage discussed below. Therefore, if the encroacher deviates and fully acquires the asset and uses it, the asset owner thereafter has to decide how much to invest in it in the next period. Given his Nash reversion strategy, he will choose the non-cooperative level of investment forever.

$$\delta \geq \frac{S_2^D - S_2^C}{S_2^D - S_2^N} = \frac{V(x)}{V(x) + \Omega - 0.25V(\hat{x})} \equiv \hat{\delta}(x), \tag{26}$$

where the expression on the RHS is the limiting case as $\varepsilon \to 0$.

Case (a): Encroacher has no bargaining power

In this case, the asset owner can make a take-it-or-leave-it offer to the encroacher. Consider an equilibrium with cooperation. Since we require $\hat{\delta} < 1$ for (26) to hold, it follows that (24) must hold with strict inequality. Therefore, a necessary condition for cooperation is $\Omega > 0.25V(\hat{x})$. Next, note that if $\Omega > 0.25V(\hat{x})$, then $\hat{\delta}$ is increasing in x. Therefore, if (26) holds for some x > x*, then it will necessarily hold for any x ≤ x*. Finally, x* = arg max [$S_1^C = V(x) - \Omega - x$]. Hence if there is cooperation the asset owner will choose x ≤ x* and if there is no cooperation, he will choose $\hat{x} <$ x*. Therefore, overinvestment is not possible.[24]

Case (b): Encroacher has some bargaining power

Suppose instead that the encroacher and asset owner bargain over the size of the transfer. The bargaining game can be captured by the maximization of the generalized Nash bargaining product,

$$M = [V(x) - \Omega - U_1^N(x)]^{\theta}[\Omega - U_2^N(x)]^{1-\theta},$$

where 0 < θ < 1, x has already been chosen to determine V(x), and M is strictly concave

[24] Another way of proving this result is by contradiction. Consider a cooperative equilibrium with overinvestment. The asset owner can *maintain* the size of the lump-sum transfer at the same level and maximize his surplus at x*. This will still ensure cooperation because V(x*) is smaller than V(x) for x > x*. Given that the size of the transfer is still the same, if the encroacher reneges he will get V(x*) + Ω which is smaller than V(x) + Ω if x > x*, so he still has no incentive to deviate from cooperation if x* is the level of investment.

in Ω.[25] Then the optimal transfer solves $\partial M/\partial\Omega = 0$ and is given by $\Omega^* = \eta V(x)$, where $\eta \equiv 0.75 - 0.5\theta \in (0.25, 0.75)$ given $0 < \theta < 1$. In this case, the transfer is a fixed proportion, η, of the value of the asset. Therefore, if the encroacher has some bargaining power, the transfer will be an increasing function of the investment in the asset.

Then $S_1^C = V(x) - \eta V(x) - x$ and (26) becomes

$$\delta \geq \frac{V(x)}{(1+\eta)V(x) - 0.25V(\hat{x})} \equiv \underline{\delta}(x) \tag{27}$$

In this case, $\underline{\delta}$ is decreasing in x.[26] Then to sustain cooperation, it may be desirable to make $\underline{\delta}$ sufficiently small by choosing $x > x^*$ and also satisfy the individual rationality constraints in (24) and (25).

To demonstrate the preceding point, let $\tilde{x}$ be the asset owner optimal level of x in a cooperative equilibrium. Define $x^{**} = \arg\max[S_1^C = V(x) - \eta V(x) - x]$. Note that $x^{**} < x^*$ given $V'(x) > 0$. Suppose $\bar{x}$ satisfies (27) with strict equality. Then $\bar{x} = \underline{\delta}^{-1}(\delta)$ and any $x < \bar{x}$ violates (27) while any $x \geq \bar{x}$ satisfies (27). Therefore, a necessary condition for overinvestment is $\bar{x} > x^*$. Given that S_1^C is maximized at x**, the asset owner will like to choose x as close as possible to x** while satisfying (24), (25), and (27). Therefore, if $\bar{x} > x^* > x^{**}$ and (24) and (25) are also satisfied at $\bar{x}$, then the optimal

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[25] Note that since x is sunk, the asset owner's threat point in the bargaining game does not include the cost of investment. It is simply his payoff in the contest. Therefore bargaining, as in Anbarci et al. (2002), takes place in the shadow of conflict. However, notice that the asset owner's individual rationality constraint in (25) is his payoff in the contest less the cost of investment. This is because to construct an equilibrium in which he cooperates in every period, he has to be guaranteed his payoff in the non-cooperative equilibrium. The same argument applies to the encroacher.

[26] When $x = 0$, the critical discount factor in (27) is equal to zero which suggests that cooperation can be sustained. But this cannot be possible because given $V(0) = 0$, the constraint in (25) is violated. Hence, (27) and, for that matter, (26) are defined for $x > 0$.

investment is $\tilde{x} = \bar{x} = \underline{\delta}^{-1}(\delta) > x^*$. Indeed, since (27) is satisfied at $\bar{x}$, this implies that $\underline{\delta}(\bar{x}) < 1$ and therefore $\eta V(\bar{x}) > 0.25V(\hat{x})$ and, for that matter, (24) is also satisfied.

If the assumptions of the preceding paragraph hold, then we know that $\partial\tilde{x}/\partial\delta < 0$. Therefore, the more patient the encroacher is, the less is the asset owner's level of investment in the cooperative equilibrium. Note that we can write $\underline{\delta}(\tilde{x}, \eta) = \delta$, where the LHS is decreasing in η using (27). Then given that $\underline{\delta}$ is also decreasing in x, it follows that when cooperation is sustained, an increase in η leads to a fall in $\tilde{x}$.

As an example, consider $\eta = 0.3$, $\delta = 0.8006$, and $W(x) = V(x) = 2x^{0.5}$. Then $x^* = 1$, $\hat{x} = 0.0625$, $S_1^N = 0.0625$, and $S_2^N = 0.125$. Let the asset owner choose $\tilde{x} = 1.5 > x^*$. Then $S_1^C = 0.2146$, $S_2^C = 0.7348$, and $\underline{\delta}(\tilde{x}) = 0.8006$. Clearly, $S_1^C > S_1^N$ and $S_2^C > S_2^N$. Note that $\underline{\delta}(x) > 0.8\,006$ for $x \in (0, \tilde{x})$. It is easy to verify that any $x > \tilde{x} = 1.5$ gives a lower value of $S_1^C = V(x) - \eta V(x) - x$ than at $x = \tilde{x}$. Therefore, given $\delta = \underline{\delta}(\tilde{x}) = 0.8006$, there is a subgame perfect equilibrium with overinvestment where the optimal investment is $\tilde{x} = 1.5 > x^* = 1$.

Note that while overinvestment can be used to induce cooperation, it is not necessarily the case that cooperation requires overinvestment. It is possible to construct a cooperative equilibrium with underinvestment, even if the transfer depends on investment. However, overinvestment can only occur in a cooperative equilibrium while underinvestment can occur in either a cooperative or a non-cooperative. Therefore, in the model, a cooperative equilibrium is necessary for overinvestment while it is not for underinvestment.

An important remark is in order. Notice that the asset owner could have chosen x* and given the encroacher a lump-sum transfer of $\tilde{\Omega} = \eta V(\tilde{x})$. This would give $\delta = \underline{\delta}(\tilde{x}) > \hat{\delta}(x^*)$, make the asset owner better off, and make the encroacher no worse off, where

$$\underline{\delta}(\tilde{x}) = \frac{V(\tilde{x})}{V(\tilde{x}) + \eta V(\tilde{x}) - 0.25V(\hat{x})},$$

$$\hat{\delta}(x^*) = \frac{V(x^*)}{V(x^*) + \eta V(\tilde{x}) - 0.25V(\hat{x})}, \text{ and}$$

$$\eta V(\tilde{x}) > 0.25V(\hat{x}).$$

By breaking the link between the transfer to the encroacher and investment, it is clear that overinvestment is not possible. Therefore, to restore the overinvestment result we need to argue that breaking the link between the transfer and investment is not possible. Notice that by arguing that the asset owner could have given the encroacher a lump-sum transfer of $\tilde{\Omega} = \eta V(\tilde{x})$, we were making the implicit assumption that we were back to the case where the asset owner could make a take-it-or-leave-it offer. Yet, if the asset owner had this power, he will not choose $\tilde{\Omega} = \eta V(\tilde{x})$. To see this, note that given $\delta = \underline{\delta}(\tilde{x}) > \hat{\delta}(x^*)$, the asset owner can sustain cooperation at x* and increase his payoff by choosing a transfer marginally smaller than $\tilde{\Omega} = \eta V(\tilde{x})$ which means that the encroacher will be worse off. Therefore, the overinvestment result still holds.

Therefore, if the encroacher has some bargaining power over how the returns from the asset should be shared or, more generally, if the transfer is increasing in the level of investment, then overinvestment is possible. I summarize the analysis in the following proposition:

Proposition 4: *If the encroacher has some bargaining power over the size of the transfer from the asset owner to him or the transfer is increasing in the level of investment, then in the in finite-period investment-cum-contest game, it is possible to construct subgame perfect equilibria with cooperation in which the owner's level of investment in the asset when property rights are incomplete is greater than his level of investment when property rights are complete.*

4.2 Further remarks

A requirement that the transfer must be linked to the proceeds of a public project could make the transfer an increasing function of investment. Such transfers may be desirable when there are institutional constraints on the nature of transfers from the asset owner to the encroacher. For example, consider a politician who can only make transfers to those who challenge his authority by investing in pork-barrel and then bargain with them over the proceeds of the project in each period.[27] This will be consistent with the logic of political survival and patronage that is documented and discussed in De Mesquita et al. (2003). Indeed, as Coate and Morris (1995) show, it may be optimal for politicians to choose inefficient forms of transfers like in-kind transfers via public projects in order

[27] It is important to note that the politician considers the investment in the pork-barrel project as a cost although he is financing it from public coffers. When Konrad (2002) applies his model to the behavior of autocrats he implicitly assumes that the politician takes the cost of investment into account. A reason why the politician may take the cost into account may be due to the moral and expected material cost of wrongdoing. For example, this makes sense if his punishment should he be out of power (e.g., by people other than his cronies) and convicted of corruption is increasing in x. Or as in Robinson and Torvik (2005), he may take this cost into account simply because every dollar spent has an opportunity cost. An example may be the distortionary cost of taxes used to finance the project.

to disguise transfers to special interests.[28] Established norms of corruption may require that the politician gets a share, $1 - \eta$, of the proceeds of the public project while his challengers get the rest. This is consistent with the commonly-held belief that kickbacks in corrupt deals are computed as some fixed proportion of government projects or contracts.[29] This may be the case because the value of the contract varies, so paying a fixed lump-sum may not make sense. However, having been a practice established over several years, it is not unreasonable to expect that this practice may still remain even if the value of contracts is expected to be constant.[30]

In some cases, the politician's main motivation may not be to make transfers to special interests. Again, suppose institutional constraints compel the politician to have surrogates who run public projects and give him his agreed-upon *share* of the proceeds of the public project. Therefore, the politician's investment decision in the project is driven by his own pecuniary motives. For example if this is a democracy with term limits and the politician is in his last term, then the nature of his transfer and investment decision are not driven by the fear of losing power. These decisions are instead driven by the fear of being prosecuted after his tenure in office.[31] This is what induces him to choose less transparent forms of transfers like public projects. Then reneging on the agreement means that his surrogates take all the proceeds from the project in a given period. The politician

[28] Of course, there is a well-known literature which argues that in the presence of moral hazard and adverse selection, in-kind transfers may be efficient.

[29] For example, Aslund (2008) mentions allegations of kickbacks of 20% to 50% on major infrastructure projects in Russia.

[30] This is analogous to the persistence of sharecropping contracts in rural and developing countries (see, for example, Allen and Lueck, 1992).

[31] Technically, though, the term limit makes the interaction between the politician and his surrogates a finitely-repeated game. However, in a democracy without term limits or in an autocracy (as in the previous example), cooperation could still be sustained if the politician is re-elected or stays in power with an exogenous probability (see Conconi and Sahuguet, 2009, Dal Bo, 2005). This means that the last period is not known with certainty. And in my model, the critical discount factor will still be decreasing in x with such an exogenous probability of staying in power. This is what is required to get the overinvestment result.

who has the exclusive right on how much should be invested in each period will then revert to the non-cooperative level of investment which, in this case, could be the minimal level of level of investment in the project. The politician and his surrogates will get nothing or a very small payoff relative to the payoff in the cooperative equilibrium.[32] As in the previous case, the politician has incomplete economic rights over the project.

In the above example, one may argue that when the politician's challengers deviate they get $S_2^D = \eta V(x) + (1-\eta)V(x) = V(x)$ not $S_2^D = (1+\eta)V(x)$. Then the critical discount factor is $\bar{\delta}(x) = (1-\eta)V(x)/[V(x) - S_2^N]$. However, it is still possible to construct equilibria with overinvestment. This is because the crucial condition that the critical discount factor, $\bar{\delta}(x)$, is also decreasing in x still holds.

5. Discussion of results

When the transfer depends on the level of investment, it acts as a distortionary tax imposed on the asset owner as opposed to a non-distortionary lump-sum tax when the transfer is independent of investment. However, given this distortion in a second-best world, it may be optimal to depart from the first-best allocation of investment even if cooperation is sustained such that there is no contest over property rights.[33]

Given that the transfer is increasing in the level of investment, cooperation is easier when there is overinvestment because it is costlier for the encroacher to renege on

[32] Generally, what matters for the analysis is that the one-period payoff from deviating from the cooperative equilibrium outweighs the payoff in the cooperative equilibrium which, in turn, outweighs the payoff in the Nash equilibrium.

[33] Subsidies for activities like R&D financed by taxes can also lead to overinvestment. However, in this case there could be overinvestment even if the subsidy was financed by a lump-sum tax. To be sure, the idea that subsidies can lead to overinvestment is obvious and such arguments are different from the argument being made here. It is not obvious that a distortionary tax can lead to overinvestment. In the present model, it is the combination of incomplete property rights and transfers that are similar to distortionary taxes which account for overinvestment.

the agreement and suffer the consequence of losing this sufficiently high transfer forever.[34] This is what gives the crucial condition that the critical discount factor must be decreasing in the level of investment. Although there is no conflict, the asset owner finds it optimal to engage in this sub-optimal investment because by inducing cooperation it saves him the cost of conflict.

The argument that overinvestment facilitates cooperation by increasing the transfer to the encroacher will not be applicable to the example of the overinvestment undertaken by squatters discussed in section 1. However, as discussed in section 4.2, it is consistent with the use of public projects by politicians as transfers aimed at holding on to political power in autocracies and democracies or as a way of transferring resources to themselves.

Unlike Robinson and Torvik (2005), it is not crucial for my result that the project must yield a negative social surplus for the politician to invest in it. In my model, the private benefit to the politician, $(1-\eta)V(x)$ must be sufficiently greater than the cost, x (i.e., $S_1^C \geq S_1^N$). Therefore, in an equilibrium with overinvestment, $V(\tilde{x}) - \tilde{x}$ is sufficiently greater than zero. However, given that the interest of the rest of society is ignored, this situation may be consistent with either a negative social surplus. Therefore, my analysis is not inconsistent with the construction of white elephants (i.e., projects with negative social surplus). However, because the asset-owner in my model (i.e., the politician in this case) does not *deliberately* invest in a project with a negative social surplus, I cannot claim that my model explains the phenomenon of white elephants in Robinson and Torvik (2005).

[34] In Skaperdas' (1992) static model in which there is no investment, cooperation is possible to sustain if the conflict success technology is sufficiently ineffective. This condition does not hold in my model.

In the case of an all-pay auction, we showed that the asset owner's investment is zero if the encroacher has a higher valuation than the asset owner. This zero investment is consistent with Smith's (2002) condition that for a high valued asset to exist in the public domain (i.e., a neglected, ill-maintained asset which tends to be common property) the encroacher must value the asset more at high values than does the owner. While Smith's (2002) intuition is correct, I have shown that his conclusion also depends on the nature of competition over property rights.

While the analysis leading to proposition 1 demonstrates that there is underinvestment in both the imperfectly discriminating contest and all-pay auction, the fact that investment in the all-pay auction is necessarily zero when the encroacher has a higher valuation but is positive when the contest is imperfectly discriminating deserves a further remark in terms of the intuition behind this difference in results. If the competition over property rights is extremely sensitive to the efforts of the contestants (e.g., all-pay auction), the battle over property rights is more likely to be very fierce. In addition, if the encroacher is stronger (i.e., has a higher valuation), then the asset owner has the incentive to minimize this extremely fierce battle by significantly reducing the value of the asset.

Proposition 3 implies that in a world of incomplete property rights an asset-owner may invest the same amount in his asset even if the nature of information in the contest over property rights is different. This suggests that when there is underinvestment, this may be driven solely by the incompleteness of property rights and not by the incompleteness of information in the contest over property rights.

6. Conclusion

Contrary to standard results in the recent literature on investment and property rights, I have shown it is possible for an asset-owner to overinvest in the asset when property rights are incomplete. As noted in the introduction, the idea that transfers or redistribution can be used to induce cooperation, when property rights are incomplete, is not new. The new result here is that the nature of transfers can lead to overinvestment.

The result of this paper does not necessarily mean that incomplete property rights are desirable because they boost investment. Like underinvestment, overinvestment also leads to a welfare loss relative to the first-best case of complete property rights. The goal of social policy ought to be the enhancement of property rights taking into account the cost of establishing such enhanced property rights. Of course, I do not mean the enhancement of the property rights of corrupt politicians.

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