Multimarket Contact in Vertically Related Markets

April 2010

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Abstract: We analyze collusion in two comparable market structures. In the first market structure only one firm is vertically integrated; there is one more independent firm in the upstream industry and another independent firm in the downstream industry. In the second market structure, there are only two vertically integrated firms that can trade among themselves in the intermediate good market. The second market structure mimics markets like the California gasoline market where firms vertically integrated through refinery, and retail markets. We rank these two market structures in terms of ease of collusion and show that while under some circumstances collusion is not possible in the market with one vertically integrated firm, collusion is possible in the market structure with two vertically integrated firms. We conclude that vertical (multimarket) contact facilitates collusion and vertical mergers suspected to lead to subsequent vertical mergers in an industry should receive higher antitrust scrutiny relative to single isolated vertical mergers.

Keywords: Multimarket; collusion; vertical integration; gasoline markets

1. Introduction

California gasoline market is well-known for its higher than national average prices and mark-ups. Even after correcting for state-specific legislative requirements, such as the CARB, this fact stands (McAfee, 2006). Thus, most authors attribute the higher-prices and margins to lack of competition due to market concentration and capacity restrictions precluding entry (Wolak, 2004). In particular, the data have been found to be consistent with some firms exercising market power (Borenstein, Bushnell, and Lewis, 2004), even though some argued that the situation is consistent with competitive markets (Energy Information Administration, 2003). Indeed, historically, a few vertically integrated oil companies have dominated both the refinery and retail levels in the California gasoline market. One characteristic of this market is the existence of several integrated companies coupled with horizontal concentration at both levels of the industry. This characteristic raises concerns about the relative likelihood of coordinated interaction especially in the presence of multimarket interaction across upstream and downstream markets. The US Merger Guidelines define coordinated effects, which is a well-known concept in merger enforcement, as "Co-ordinated interaction is comprised of actions by a group of firms that is profitable for each of them only as a result of the accommodating reactions of the others. This behavior includes tacit or express collusion…" (see http:// ec.europa.eu/dgs/competition/economist/delamano2.pdf). Not surprisingly given the concentration in the California gasoline market, there is also a concern for coordinated effects in merger enforcement. In particular, the Federal Trade Commission has objected to several mergers in this particular market based on coordinated effects theories (see FTC, 2003, http://www.ftc.gov/ftc/oilgas/charts/merger_enforce_actions.pdf). In this paper, we emphasize that there is one aspect of a market such as the gasoline market in California that requires further attention than would be given to a standard concentrated market in a horizontal context. That is, there is the possibility of multimarket interaction of vertically integrated firms embedded in a market composed of vertically related levels. Multimarket contact is generally known to facilitate collusion, but the extension of this argument to vertically related levels is not trivial as we demonstrate in this paper. This paper is also relevant to antitrust policy concerning vertical mergers where a vertical merger that may lead to a subsequent one or a vertical merger that results in higher concentration in both the upstream and the downstream markets.

Although integrated, the oil companies in California regularly trade the refined gasoline among themselves, leading to differing market shares between the refined gasoline (intermediate good) and retail gasoline (final good) markets. Thus, these companies contact and compete with each other in multiple markets (McAfee and Hendricks, 2009), and, as is well-known, apart from market concentration within a given market, multi-market contact in general further facilitates tacit collusion by allocating market power across the participants according to their relative efficiencies or spheres of influence in the product space (Bernheim and Whinston, 1990). Note, however, that the vertically related nature of refined and retail gasoline markets constitutes a special form of multimarket contact, which we call multilevel contact, and this type of contact has never been formally modeled (McAfee, 2003). In particular, since these markets are inherently (vertically) related, Bernheim and Whinston's (1990) seminal paper, which finds that multimarket contact generally facilitates collusion, may not necessarily apply to them, and even if the argument applies, characterization of the environment where collusion is facilitated is critical for ensuing antitrust policy. Although vertical mergers have been extensively studied, there are still open questions (see Higgins (2009) for an excellent review). While we point out the California gasoline market as an example, our paper applies to any industry with a high concentration of vertically integrated firms.

In modeling multilevel collusion, one needs to play simultaneous attention to collusion in all the vertically related markets. As Nocke and White (2007), who study upstream collusion in their paper, put it, "One would of course like to know how vertical integration might facilitate collusion between firms at each level of the vertical hierarchy; this is an open question…" In this paper, we consider all levels of the industry, and we provide a model of multilevel collusion and collusion in a market structure with a single vertically integrated firm. We show that multilevel collusion facilitates collusion and point out what specifics need to be worked out in order to extend the Bernheim and Whinston (1990) result to this setting. Our result, then, suggests a more aggressive push towards vertical divestitures in vertically related levels as part of merger enforcement. For example, in the case of gasoline in California, divestiture of retail gasoline has been proposed by Wolak (2004), who conditions this on high costs of concentration to consumers. Our results also support a dynamic view of merger enforcement in that a given vertical merger to be followed by several others may be disproportionately more harmful then an isolated one or the first one (McAfee, 2006).

In the literature, to our knowledge, multimarket collusion in vertically related markets, in which there are cost- and demand- based linkages across markets, has not been investigated. Modeling multilevel collusion is complicated due to the inherent relationships between the markets. One challenge is the benchmark model to compare the structures with several vertically integrated firms. Also, one needs to cover a variety of possibilities especially when modeling deviation, e.g., an independent upstream firm can deviate from collusion or a downstream firm, simultaneously or sequentially. Notwithstanding these challenges we provide a reasonable collusion model and fairly general conditions on model parameters under which collusion is sustainable only with more than one vertically integrated firm.

Vertical mergers are increasingly found to have anti-competitive elements, mostly under the umbrella of post-Chicago theories. The "raising rivals' costs" and "facilitating collusion" theories are two strands of this literature. In this paper, we find results that combine both strands of this literature, however our focus is on the latter. Although the topic of vertical mergers may seem to have been exhausted at first sight (see comments of Higgins (2009)), the numerosity of possibilities in the vertical structure seems to continuously lead to new models (See Higgins (2009), Nocke and White (2007), Normann (forthcoming), Chen (2001), Ayar (2008)). Collusion in vertical settings is a fairly important topic exactly because of the subtleties and *ad hoc* nature of these settings. Thus, we contribute to the multiplicity of these models to shed more light on this important economic and antitrust issue.

To obtain our results, we use the repeated games technique and the Cournot model when there is competition (such as the punishment and deviation phases). The usage of the repeated games technique is standard in collusion settings and we also want to make our results comparable to those of Bernheim and Whinston (1990) and to the literature that stems from that paper. The usage of Cournot modeling is, first, due to our interest in examining this question in a homogeneous market setting in order to model commodity products such as gasoline. Second, the Cournot model is more useful in modeling market power in either the upstream or the downstream markets in terms of the margins that it generates as well as some of the other relevant aspects of the industry such as intra-industry trade, where the Bertrand model falls short (see, for example, McAfee and Hendricks, 2009). Finally, Cournot model better approximates conscious parallelism that is one of the main concerns of the antitrust authorities (McAfee, 2006).

We first model the case in that there is only one integrated firm and investigate optimal collusion. In particular, our assumption is that the integrated firm can only sell the intermediate goods at a price that is equal to the cost of the (less efficient) unintegrated upstream firm. This assumption is not critical for our results, and the efficient firm's leadership replicates the most efficient collusion possible, i.e., where there is no intermediate market separating upstream and downstream markets (optimal collusion). As a result, we show that such collusion is not preferred to Cournot competition by the single vertically integrated firm, and so this precludes collusion. We provide the conditions where the only integrated firm elects to withdraw from the market a la Ordover, Salop, and Saloner (1990).

Two recent papers that tackle similar issues are discussed next. First, Nocke and White (2007) analyze the effects of vertical integration upstream collusion and they use a two-part tariff in pricing. When investigating only upstream collusion Nocke and White (2007) also find a similar "punishment" effect, which is overweighed by the "outlets" effect. Nocke and White (2007) also have a section on multiple vertical integrations, which they simply extend their findings via comparative statics of the model with one vertical integration. Second, Normann (forthcoming) studies upstream collusion in the same setting as Nocke and White (2007) except that he uses linear pricing as we also do. Normann (forthcoming) however uses Bertrand competition which greatly eliminates any strategic involvement of upstream firms. Our paper is different from these two papers in that intermediate market pricing is set by Cournot competition and hence our moving from one vertically integrated to two such firms does not have to parallel what would be suggested by running comparative statics on the case with one vertically integrated firm.

As a result, we show that under certain circumstances collusion is possible with two integrated firms but not with one. This establishes that multilevel contact facilitates collusion. The structure of the paper is as follows: In the next section, we discuss our model and in the third and fourth sections we present our results. In the final section we conclude with a discussion of the policy implications of our results.

2. Model

In our model there are two vertically related levels, upstream (like refining crude oil) and downstream (like retailing gasoline), and correspondingly two markets, the intermediate and the final good markets. There are two firms in each of the upstream and downstream levels. We denote the two upstream firms with U1 and U2 and the two downstream firms with D1 and D2. To denote a vertically integrated firm formed from the integration of Ui and Di we use the notation Ui–Di, i=1,2. We study collusion possibilities in two different cases based on the number of integrated firms, which is denoted by $m \in \{1,2\}$:

Case 1: Single Vertical Integration (*m*=1)
Case 2: Multilevel Contact (*m*=2)

The demand for the final good is exogenously given by $Q_f = a - P_f$, where we normalize the slope of the demand to unity. The upstream firms U1 and U2 have asymmetric constant marginal costs, which satisfy $0 < c_1 < c_2 < a$ and there are no fixed costs of production (The asymmetric cost assumption serves as a tie-breaking rule in collusive profit sharing. Our results trivially extend to symmetric marginal costs.). For simplicity, we assume a fixed proportions technology with one-to-one transformation between the input and the output.

We use the infinitely repeated games technique and focus on symmetric subgame perfect equilibria. For each case $m \in \{1,2\}$ we proceed in order through collusion, deviation, and punishment stages. In collusion stages, we assume that collusion is set to yield the highest collusive payoff by setting prices to the monopoly price. We assume that production is made by the lowest-cost producer when at least one firm is integrated so that joint-profits are maximized (when no firms are integrated we assume that collusions are independent in the upstream and downstream, as opposed to all four firms coming together at the optimal collusion.). We use one consistent sharing rule to divide the optimal collusive profits across all cases. In deviation and punishment stages, we solve Cournot-style games, assuming that firms' decision variables are their production quantities and transactions between independent firms always take place through the intermediate goods market. In the following sections, we cover our two cases.

3. Collusion Analysis with a Single Vertically Integrated Firm (*m* =1)

3.1. Collusion Stage with a Single Vertically Integrated Firm

There are only three distinct firms in this case. The only vertically integrated firm is U1–D1, and U2 and D2 operate in the upstream and downstream markets, respectively (the extension of the analysis to the case where U2-D2 is the only integrated firm is straightforward. In that case collusion profits for U2-D2 would remain unaltered, however deviation profits would decrease.). In the collusive stage, the low-cost firm U1–D1 sells D2 all the intermediate goods it needs in exchange for a unit price of c_2, which is the marginal production cost c_2 of U2. This assumption is consistent with optimal collusion maximizing joint industry profits. With such a low price, we assume that U2, with cost c_2, is foreclosed from the market during collusion. Finally, U1-D1 splits the downstream monopoly quantity equally with D2, so we use the 50-50 production sharing rule.

The monopoly output to be sold at the downstream market with equal shares is computed using the demand curve $Q_f = a - P_f$ and the cost c_1 (leading to industry profit maximization output hence to optimal collusion), which is $(a-c_1)/2$. As mentioned earlier, each firm equally shares the monopoly output, i.e., $(a-c_1)/4$, at the monopoly price $(a+c_1)/2$. Assuming D2 pays c_2 to U1-D1 for each unit, which U1-D1 produces at a cost of c_1, the implied profits for the three firms are readily computed:

$$\Pi_1^{col,m=1} = ((a+c_1)/2 - c_1)(a-c_1)/4 + (c_2 - c_1)(a-c_1)/4$$

$$\Pi_2^{col,m=1} = 0$$
$$\Pi_3^{col,m=1} = ((a+c_1)/2 - c_1)(a-c_1)/4 - (c_2 - c_1)(a-c_1)/4,$$

and simplified to

$\Pi_{\text{U1-D1}}{}^{col,m=1} = (a-c_1)^2/8 + (a-c_1)(c_2-c_1)/4$, $\Pi_{\text{U2}}{}^{col,m=1} = 0$, and
$\Pi_{\text{D2}}{}^{col,m=1} = (a-c_1)^2/8 - (a-c_1)(c_2-c_1)/4$

Once again, to check the participation constraint (individual rationality of participation), we compare these collusive profits with profits from Cournot competition where, in theory, both U1-D1 and U2 can produce and there is an intermediate market. There is no individual rationality of participation concern for U2 because by construction it is excluded from collusion. Indeed, in this case also we show that collusive profits from collusion are lower than those in punishment. We skip the discussion of deviation profits since deviation profits do not matter for collusion as explained in the next section on punishment.

3.2. Punishment Stage with a Single Vertically Integrated Firm (*m* =1)

We first establish in Proposition 1 below that in the case of punishment U1-D1, the only integrated firm, does not participate in the intermediate good market in equilibrium.

In the punishment phase of case m=1, the independent upstream firm U2 produces the intermediate good at a cost of $c_2 > c_1$ and sells at the intermediate good price P_I to D2, where P_I is determined in the market. Firm D2 is the only independent downstream firm, and it engages in Cournot competition with U1-D1 in the downstream market. Proposition 1 shows that in equilibrium firm U1-D1 does not sell inputs to D2 and also establishes the impossibility of collusion when $m=1$.

Proposition 1. Assume that in the punishment phase the downstream firms, U1-D1 and D2, have to compete a la Cournot among themselves and the intermediate good market remains in operation with U2 supplying D2. If U1-D1 and D2 collude by sharing downstream sales equally and in exchange having D2 pay c_2 to U1-D1, then such collusion is not possible because U1-D1's profits at the punishment stage are higher and it immediately deviates.

Proof of Proposition 1.

To obtain our result, we solve for equilibria of two games and compare U1-D1's profits (one can think of these two different games as a unified game by introducing a first stage where U1-D1 decides which one to play). In Game 1, U1-D1 does not participate in the intermediate goods market, in Game 2 it does. Then, we use the profits of U1-D1 from Game 1 and compare it with the profits of Game 2. We show below that profits are higher in equilibrium when U1-D1 does not participate.

Below is the setup for Game 1:

Table 1. One stage punishment game (Game 1) with single vertical integration (*m*=1)

Player	Strategic Variable ($\in R^+$)
U1-D1	q_1
U2	x_2
D2	q_2

This *simultaneous* Cournot game is played by U1–D1, U2, and D2, given downstream demand and an intermediate market. The equilibrium for Game 1 can be found as follows. Profits of the firms from downstream sales are:

$\Pi_{U1-D1} = (a - q_{U1-D1} - q_{D2} - c_1)q_{U1-D1}$ and $\Pi_{D2} = (a - q_{U1-D1} - q_{D2} - P_I)q_{D2}$.

Solving for the Cournot quantities $q_{U1-D1} = q_1^* = (a - 2c_1 + P_I)/3$, $q_{D2} = q_3^* = (a + c_1 - 2P_I)/3$.

Now we proceed to solve for P_I from the equation of q_{D2} because as long as $P_I > c_1$ the firm U1-D1 always purchases the inputs from itself: $P_I = (a + c_1 - 3q_3^*)/2$.

Recalling our assumption on one-to-one transformation, note that $q_2^* = q_3^*$, and hence the demand for the intermediate goods becomes $P_I = (a + c_1 - 3q_2^*)/2$. Firm U2 maximizes its profit: $\max\,(P_I - c_2)q_2 = ((a + c_1 - 3q_2)/2 - c_2)q_2$.

The maximizing quantity is $q_2^* = (a + c_1 - 2c_2)/6 = q_3^*$. Plugging it into the other expressions we have,

$q_1^* = (5a - 7c_1 + 2c_2)/12$, $P_I^* = (a + c_1 + 2c_2)/4$, and $P_f^* = (5a + 5c_1 + 2c_2)/12$.

Note that $P_I > c_1$. Also, $P_f^* > c_2$ if and only if $a > 2c_2 - c_1 = c_2 + (c_2 - c_1)$, our earlier assumption. Thus, U1-D1's profit from punishment modeled as Game 1 equals

$\pi_1^{pun} = (P_f^* - c_1)q_1^* = ((5a - 7c_1 + 2c_2)/12)^2$.

Next we move on the Game 2:

Table 2. Another one stage punishment game (Game 2) with single vertical integration (*m*=1)

Player	Strategic Variable ($\in R^+$)
U1-D1	$x_{U1\text{-}D1}, q_{U1\text{-}D1}$
U2	x_{U2}
D2	q_{D2}

Firms U1-D1 and D2 maximize their profit functions with respect to downstream quantities (note that we are not solving this game via backward induction, but in two steps)

$\max_{q_{U1-D1}} \Pi_{U1-D1} = (a - q_{U1-D1} - q_{D2} - c_1)q_{U1-D1} + (P_I - c_1)x_{U1-D1}$

$\max_{q_{D2}} \Pi_{D2} = (a - q_{U1-D1} - q_{D2} - P_I)q_{D2}$, which yield

$q_{U1-D1} = (a - 2c_1 + P_I)/3$, and $q_{D2} = (a + c_1 - 2P_I)/3$.

Since U1-D1 purchases the inputs from itself, only q_{D2} determines the inverse demand for firm U2 and firm U1-D1: $P_I = (a + c_1 - 3(x_{U1-D1} + x_{U2}))/2$,

where $x_{U1-D1} + x_{U2} = q_{D2}$, so U1-D1 and U2 are both selling to D2. Incorporating this market clearing condition to the profit functions of U1-D1 and U2 we have

$\Pi_{U1-D1} = ((a + c_1 - 3(x_{U1-D1} + x_{U2})/2) - c_1)x_{U1-D1}$

$\Pi_{D2} = ((a + c_1 - 3(x_{U1-D1} + x_{U2})/2) - c_2)x_{U2}$

Maximizing each profit function with respect to quantities, we have

$x_{U1-D1} = (a - 3c_1 + 2c_2)/9$, and $x_{U2} = (a + 3c_1 - 4c_2)/9$. Hence $P_I = (a + 3c_1 + 2c_2)/6$, $q_{U1-D1} = (7a - 9c_1 + 2c_2)/18$, $q_{D2} = 2(a - c_2)/9$, and $P_f = (7a + 2c_2 + 9c_1)/18$.

Note that $P_I < P_f$ holds since $a > c_2$. The profits are,

$\Pi_{U1-D1} = (7a + 2c_2 - 9c_1)^2/324 + (a + 2c_2 - 3c_1)^2/54$, $\Pi_{D2} = 2(a - c_2)^2/81$ and

$\Pi_{U2} = (a - 4c_2 + 3c_1)^2/54$.

A comparison of equilibrium profits for U1-D1 from Game 1 and Game 2 reveals that Game 1 profits are higher, i.e.

$((5a - 7c_1 + 2c_2)/12)^2 > (7a + 2c_2 - 9c_1)^2/324 + (a + 2c_2 - 3c_1)^2/54$

So U1-D1 does not participate in the intermediate good market, and the punishment game is Game 1. Finally, when $a > 2c_2 - c_1$.

Collusive profit of U1-D1 $= (a - c_1)^2/8 + (a - c_1)(c_2 - c_1)/4 < ((5a - 7c_1 + 2c_2)/12)^2 =$ punishment profit of U1-D1.

So with this sharing rule, collusion is impossible because U1-D1 will defect. □

We provide the intuition next. First, the equilibrium final good price $P_f^* = (5a + 5c_1 + 2c_2)/12$ applies to all the quantities sold by firm 1, whereas in collusion U1-D1 was selling some of its goods to D2 at a low price of c_2 in our setting. Second, when firm 1 competes in Cournot fashion with D2, it has a great advantage due to the arising cost structure: $P_I > c_2 > c_1$, provided the condition $a > 2c_2 - c_1$ holds. In collusion, U1-D1 has to sacrifice more profits. Thus, $P_f^* = (5a + 5c_1 + 2c_2)/12$ is "not too low" compared to the collusive price. Third, obviously, the expansion in output of U1-D1 due to Cournot competition with relatively high equilibrium price increases the profits of firm 1 in this "punishment" phase. Simply put, firm 1 has nothing to gain from such collusion even though D2 prefers to collude whenever $a < 2c_2 - c_1$.

Next we study the case m = 2, which corresponds to multilevel contact.

4. Multilevel Contact (*m*=2)

In this case there two integrated firms and no others. These firms are denoted by U1-D1 and U2-D2. There is still an intermediate market in deviation and punishment phases due to the cost asymmetry.

4.1. Collusive Phase (*m*=2)

In this case, firms engage in optimal collusion, i.e., maximize industry profits by producing the monopoly output corresponding to the lowest cost upstream firm (U1-D1 producing at cost c_1). Also, as in the case *m*=1, all firms make equal sales at the downstream level. Only U1-D1 produces the whole industry output at the upstream level and sells an equal share to U2-D2 at a side-payment of c_2. Since there are two entities participation constraint is equivalent to sustainability of collusion, which we show is the case. Now we proceed to solve the model under collusion. The collusive profits are the same as in the case *m*=1 because we had excluded U2 from collusion in the case of *m*=1 (but U2 is an active producer and Cournot competitor during deviation and punishment phases when *m*=1 and when *m*=2).

$$\Pi_{U1-D1}{}^{col,m=2} = \Pi_{U1-D1}{}^{col,m=1} = (a-c_1)(a-3c_1+2c_2)/8$$

$$\Pi_{U2-D2}{}^{col,m=2} = \Pi_{D2}{}^{col,m=1} = (a-c_1)(a+c_1-2c_2)/8$$

Next we move forward with the analysis of deviation and punishment.

4.2. Deviation Phase (*m*=2)

We assume that only one player deviates at a time via hidden production, which is observed only after the sales. The deviation profit for U1-D1 is the same as that in *m*=1:

$$\Pi_{U1-D1}{}^{dev,m=2} = \Pi_{U1-D1}{}^{dev,m=1} = (a-c_1)(9a+16c_2-25c_1)/64.$$

On the other hand, the optimal deviation profit for U2-D2 is different because when *m*=1, D2 must buy from U2, who is the only source, so U2 charges a higher price than c_2. The deviation profit for U2-D2 is computed as, (in the Appendix we show a derivation of the profit expressions): $\Pi_j^{dev} = (3a+c_1-4c_2)^2/64$.

4.3. Punishment Phase (*m*=2)

The model in this section is a simultaneous-move game where, given downstream demand, each integrated firm determines its upstream production level x_i and downstream sales q_i

subject to the equilibrium constraint $Q_f^* = \sum_i x_i$ (see McAfee and Hendricks, 2009, for a similar model). Total sales equal total production and hence the intermediate market clears.

Player	Strategic Variable ($\in R^+$)
U1-D1	$x_{U1\text{-}D1}, q_{U1\text{-}D1}$
U2-D2	$x_{U2\text{-}D2}, q_{U2\text{-}D2}$

This punishment model is suitable in many industries, including the oil industry, in which we observe spot markets. Moreover, this assumption helps us to purely abstract from any form of limited or partial vertical integration such as contracts.

Let q_i be downstream (e.g. retail) sales of firm *i*, x_i be upstream (e.g. refinery) production of firm *i*, and P_I be the price of the intermediate good (e.g. refined gasoline). To find the pure strategy Nash equilibrium, (we write the problem for general case. To calculate the prices and outputs replace *n*=2 and *i*=1,2.), we solve the first order conditions for q_i and x_i subject to the equilibrium constraint. The profit function for firm *i* is (*i*=1,2)

$$\Pi_i^c(.) = (a - Q_f)q_i - P_I(q_i - x_i) - c_i x_i = (P_f - P_I)q_i + (P_I - c_i)x_i,$$

where the profit from the sale of quantity q_i is added to that obtained from the production quantity x_i. The first order necessary conditions lead to

$q_i^* = a - Q_f^* - P_I$ and $Q_f^* = n(a - P_I)/(n+1)$, where *n*=2.

Since $Q_f^* = \sum_i x_i$, we have $P_I = a - (n+1)\sum_i x_i / n$, (indicating a more inelastic demand for the intermediate good). The profit function for firm *i* in the Cournot stage can be rewritten as,

$$\Pi_i^c(.) = (P_f^* - P_I)q_i^* + (P_I - c_i)x_i = (a - P_I - \sum_i x_i)(a - P_I)/(n+1) + (P_I - c_i)x_i,$$

where we employ that $q_i^* = (a - P_I)/(n+1)$ and $P_f^* = a - \sum_i x_i$. Then the profit as a function of the upstream quantities is $\Pi_i^c(.) = (\sum_i x_i / n)^2 + [n(a-c_i) - (n+1)\sum_i x_i]x_i / n$.

The first order necessary conditions of the profit function provide,

$$X^* = \sum_i x_i^* = \frac{n(na-c)}{(n+1)^2 - 2} = (4a - 2c_1 - 2c_2)/7,$$

where $c = \sum_i c_i$, $x_i^* = \dfrac{a(n+1)n + c(n^2+n-2) - c_i((n+1)^2-2)n}{(n+1)((n+1)^2-2)}$.

We can now calculate the equilibrium intermediate good price, $P_I = (a(n-1) + c(n+1))/((n+1)^2 - 2) = (a + 3c_1 + 3c_2)/7$.

Thus, the optimal profit level for each player *i* in the punishment phase is,

$\Pi_i^c = (X^*/n)^2 + [(a-c_i) - (n+1)X^*/n]x_i^*$,

where x_i^*, X^* are defined as above.

The profit expression is,

$\Pi_i^c(.) = (P_f^* - P_I)q_i^* + (P_I - c_i)x_i = (a - P_I - \sum_i x_i)(a - P_I)/(n+1) + (P_I - c_i)x_i$, where

$P_f^* = (3a + 2c_1 + 2c_2)/7$, $P_I^* = (a + 3c_1 + 3c_2)/7$, $q_{U1-D1} = q_{U2-D2} = (2a - c_1 - c_2)/7$,

$x_{U1-D1} = (6a - 10c_1 + 4c_2)/21$, $x_{U2-D2} = (6a + 4c_1 - 10c_2)/21$.

Note that efficient firms are net sellers and inefficient firms are net buyers of the intermediate good in the punishment phase where Cournot style competition prevails with full multilevel contact. This can be calculated by noting that $q_i^* = \dfrac{(na-c)}{(n+1)^2-2}$, and the difference between sales and production is $q_i^* - x_i^* = \dfrac{c_i n - c}{n+1}$, which takes either sign. Specifically $q_2^* > x_2^*$ and $q_1^* < x_1^*$ hold since $c_2 > c_1$.

Comparison of the traded amounts:

$q_1^{pun,m=2} - x_1^{pun,m=2} = (c_1 - c_2)/3 < 0$, then U1-D1 is net seller of intermediate good.

$q_2^{pun,m=2} - x_2^{pun,m=2} = (c_2 - c_1)/3 > 0$, then U2-D2 is net buyer of intermediate good.

4.4. The Possibility of Collusion (m=2)

In the previous sections we show that collusion is impossible when $m < 2$. In this section, we show that collusion is sustainable when $m = 2$ under the same assumptions and comparable structures. Our method at this point onwards is fairly standard. Since we can readily compute the profits from collusion, deviation, and punishment phases for each firm, a cutoff discount factor that ensures collusion follows for each firm. The ultimate discount factor to sustain collusion is the maximum of these cutoff discount factors.

Proposition 2. Collusion is possible when m=2.
Proof. See the Appendix.

In the proof, where we normalize $c_1 = 0$ and assume that $a \geq 8c_2$ for illustrative purposes, but these sufficiency conditions can be made much weaker, making the domain of the collusion possibility result much larger. These sufficiency conditions render a discount rate strictly between zero and one in the case of full multilevel contact (m=2). Thus, under very general conditions on model parameters, collusion is only sustainable when $m = 2$ but not when $m < 2$. So under our assumptions two vertically integrated firms are needed for collusion to be sustained.

5. Conclusion

We consider optimal collusion possibilities in a vertically related industry that is composed of one upstream and one downstream component. We compare two cases. In our first case, there is one vertically integrated firm, one independent upstream firm, and one independent downstream firm. We show that under equal collusive profit sharing rule when the integrated firm colludes with the independent downstream firm and forecloses the independent upstream firm, collusion is impossible. In our second case, there are two vertically integrated firms, and all production is done by the lowest cost firm to be consistent with optimal collusion as in the first case, and the higher cost firm receives side payments. As a result we show that collusion is possible in the second case only.

Our results show that the number of vertically integrated firms is a critical decision variable for an antitrust authority in deciding whether to approve a vertical merger. The FTC's actions in the petroleum industry demonstrate that since 1981 every merger that FTC took action upon is accompanied by another one within one year of the action. In the last decade, there were two mergers or attempts in 2001, 2002, 2004, and 2007 and three in 2005. This record favors a dynamic view of mergers, where merger decision is considered as strategically made in anticipation of other mergers. Our current paper provides a model of how vertical mergers can be endogenized.

APPENDIX

Deriving profit expressions in Section 4.2 Deviation (m=2).

The deviation profits for firm 1 are computed as

$$\Pi_1^{dev}(.) = (a - Q_f^* - z_1)(z_1 + Q_f^*/n) + c_2 Q_f^*(n-1)/n - c_1(Q_f^* + z_1),$$

where z_1 is the hidden production level for firm 1. Profit maximization deviation level is solved as $z_1^* = (n-1)(a-c_1)/4n$ leading to

$$\Pi_1^{dev} = \frac{(a-c_1)[a(1+n)^2 + 8c_2 n(n-1) - c_1(1-3n)^2]}{16n^2}.$$

Similarly, the deviation profits for other firms ($j \neq 1$) become (replace j = 2 and n = 2)

$$\Pi_j^{dev}(.) = (a - Q_f - z_j)(z_j + (a-c_1)/2n) - c_2(a-c_1)/2n - c_j z_j.$$

Profit maximizing hidden production level is

$$z_j^* = [c_1(n+1) + a(n-1) - 2nc_j]/4n.$$

Then the profit for firm j can be calculated as,

$$\Pi_j^{dev} = \frac{(c_1(n-1))^2 + (2c_j n)^2 + (a(1+n))^2 - 4c_1 n(-2c_2 + c_j(1+n)) + 2a(2n(-2c_2 + c_j(1-n)) + c_1(n^2-1))}{16n^2}$$

Proof of Proposition 2.

First we present the expressions for the cutoffs above which collusion can be sustained. The collusion condition for firm i is $\frac{\Pi_i^{col}}{1-\delta_i} \geq \Pi_i^{dev} + \delta_i \frac{\Pi_i^c}{1-\delta_i}$. It implies, for firm 1, that

$$\delta_1(n,a,c_1,c_2) \geq \frac{d_1}{d_2+d_3+d_4},$$

where $d_1 = -(a-c_1)^2(1+n)(n^3+n^2-3n+1)^2$,

$d_2 = 16c^2n^2(n^3+2n^2-1)+(c_1(n^2+2n-1))^2(7n^3-3n^2+5n-1)-a^2(n-1)^2(n^5+9n^4+2n^3-6n^2+n+1)$

$d_3 = 8c_1n(n^2+2n-1)(-4cn(n^2+n-1)+c_2(n^4+2n^3-2n^2-2n+1))$

$d_4 = 2a(n-1)[c_1(5n^6+12n^5+n^4-5n^2+4n-1)-4n(-4cn(n^2+n-1)+c_2(1+n)(n^2+2n-1)^2)]$

and $c = \sum_i c_i$.

Similarly, collusion will be maintained by firms $j \neq 1$, if and only if,

$$\delta_j(n,a,c_1,c_2,c_j) \geq \frac{e_1}{e_2+e_3+e_4},$$

where $e_1 = (1+n)(c_1+a(n-1)+nc_1-2nc_j)^2(n^2+2n-1)^2$,

$e_2 = -4nc_1(1+n)(-2c_2+c_j(1+n))(n^2+2n-1)^2+c_1^{\,2}(n+1)(n^3+n^2-3n+1)^2$
$\quad +a^2(n-1)^2(n^5+9n^4+2n^3-6n^2+n+1)$

$e_3 = -4n^2[c_j^{\,2}(3n-1)(n^2+2n-1)^2+4c^2(n^3+2n^2-1)-8cc_j(n^4+3n^3-3n+1)]$

$e_4 = 2a[c_1(n-1)(n^3+3n^2+n-1)^2-2n[2c_2(n+1)(n^2+2n-1)^2$
$\quad +8cn(n^3-2n+1)+c_j(n^6-4n^5-15n^4-n^2+4n-1)]]$

Collusion by all firms is possible if and only if the actual discount factor is greater than δ^{mlc}, where $\delta^{mlc} = \max(\delta_1,\delta_j)_{j>1}$. Now Let the difference of the discount factors in multilevel contact be $\Delta = \delta_2 - \delta_1$. Then for n =2,

$$\Delta = \frac{4704(c2-c1)(29a^2+186ac_1-95c_1^{\,2}-244ac_2+4c_1c_2+120c_2^{\,2})}{(171a-271c_1+100c_2)(171a^2-2054ac_1+923c_1^{\,2}+1712ac_2+208c_1c_2-960c_2^{\,2})}$$

. There is no further compact representation of this term. To get the sign of this term, normalize the cost c_1 to zero so that the comparison is rendered. Observe that, $\delta_2\big|_{c_1=0} = \dfrac{147(a-4c_2)}{171a+100c_2} > 0$ if and only if $a > 4c_2$, and $\delta_1\big|_{c_1=0} = \dfrac{147a^2}{171a^2+1712ac_2-960c_2^{\,2}} > 0$ if and only if

$171a^2 + 1712ac_2 - 960{c_2}^2 > 0$. Then, obviously,

$\Delta\big|_{c_1=0} = \dfrac{4704c_2(29a^2 - 244ac_2 + 120{c_2}^2)}{(171a + 100c_2)(171a^2 + 1712ac_2 - 960{c_2}^2)} > 0$, by using above two inequalities and $a \geq 8c_2$. Note that whenever $a \geq 8c_2$ holds, then $\delta_1\big|_{c_1=0}$, $\delta_2\big|_{c_1=0}$, $\Delta\big|_{c_1=0} \in (0,1)$. □

Acknowledgements:

The views expressed here are ours and should not be construed as representing the positions of other experts at LECG or other institutions that any of the authors are affiliated with. We thank Ömer Faruk Baykal, Tim Brennan, Ken Hendricks, Preston McAfee, Richard Shin, and Mike Hoy for helpful discussions and comments. We also thank the participants at the International Industrial Organization Conference for insightful comments especially to discussants Jeroen Hinloopen and Yuk-Fai Fong.

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