# A Large Trader in Bubbles and Crashes: An Application to Currency Attacks

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January 29, 2010

**Abstract**

Abreu and Brunnermeier (2003) study stock market bubbles and crashes in a dynamic model with a continuum of rational small traders. We introduce a large trader into their model and apply it to currency attacks. In an attack against a fixed exchange rate regime with a gradually overvaluing currency, traders lack common knowledge about the time when the overvaluation starts. Meanwhile, they need to coordinate to break a peg. In such a setup, both the inability of traders to synchronize their attack and their incentive to time the collapse of the regime lead to the persistent overvaluation of the currency. We find that the presence of a large trader with perfect information will accelerate the collapse of the regime and alleviate currency overvaluation. However, if a large trader has incomplete information, the presence of a large trader may accelerate or delay the collapse of the regime ex post, depending on the size of his wealth and the precision of his information. More specifically, we find that a large trader with both a large amount of wealth and very noisy information can greatly delay the collapse of the regime ex post. Moreover, we find that the presence of a large trader with incomplete information can greatly increase the unpredictability about the time when the regime collapses, implying the difficulty for traders to time the collapse.

Keywords: Large Trader, Bubbles and Crashes, Currency Attack

JEL Classification: D80, F31, G12

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# 1 Introduction

Asset bubbles where asset prices deviate from fundamental values, and their subsequent crashes, can inflict great damage on an economy through various channels of resource misallocation. A large body of economic literature has discussed this issue, especially since the current financial crisis. One strand of the literature explains why asset bubbles and crashes can exist even in the presence of wealth constrained *rational* arbitrageurs. This argument challenges the Efficient Market Hypothesis by assuming that there is a mass of behavioral traders who are unaware of the bubble, but potential rational arbitragers become only aware of the bubble sequentially until they, and their combined wealth, can have sufficient impact to burst the bubble. The paper developed by Abreu and Brunnermeier (2003) is one of the seminal theoretical papers in this literature. In their model, rational arbitrageurs in a market (they analyze stock markets as an example) become aware of an asset bubble sequentially. Due to the lack of common knowledge about the bubble, and need for coordination to burst the bubble, the bubble will be persistent and its bursting time depends on the incentives of the arbitrageurs to " ride the bubble," as opposed to incentives to preempt other arbitrageurs in selling the asset.

In Abreu and Brunnermeier (2003), rational arbitrageurs are negligibly small and identical ex ante. This model does not address the empirically important case where there is a large arbitrageur and the mass of small arbitrageurs who must time their bursting of the bubble. Large traders (e.g., hedge funds and banks) are an important group of market participants in asset markets. For example, large traders such as George Soros have been alleged to have played an active role in currency attacks against fixed exchange rate regimes in foreign exchange markets. They include the attack against British pound in 1992, and the attacks against East Asian currencies in 1997 and 1998. Large traders launch currency attacks by employing short positions. They try to influence market sentiment by publicly announcing their short positions and beliefs that devaluation is inevitable. This causes herding among small traders, and/or deters contrarians from taking opposite positions. It seems that the presence of large traders facilitates coordination among traders and increases financial instability in the attacked currencies, specially in small economy currencies. This is sometimes called the "big elephants in small ponds" effect.

To analyze this type of market, we develop a model to study the role of a large trader in a currency attack. The model is based on the analysis in Abreu and Brunnermeier (2003). Similar to their model, we assume that a currency begins to be overvalued in a fixed exchange rate regime after a certain time. Traders have dispersed opinions in the sense that they only become aware of the overvaluation sequentially. In addition, we assume that the fixed exchange rate regime will collapse only when attacking pressure reaches a threshold level. This assumption captures the main feature of currency attacks: there is a necessity for coordination among traders to break a currency peg. This coordination feature is emphasized in Obstfeld (1996) and other currency attack models (see especially Morris and Shin (1998)). In our setup, traders try to choose the optimal time to launch their attack, driven by two competing incentives: first, the incentive to "ride the overvaluation;" and second, the incentive to preempt other traders. The traders' incentive to "ride the overvaluation" stems from two sources in our model: first, they can reap higher benefits from the devaluation if the overvaluation lasts longer. Second, if they time their attack more precisely, they will save on attacking costs. The trader attempts to preempt other traders, because only the trader attacking early will gain from the collapse of the regime. The late traders will gain nothing.

Abreu and Brunnermeier (2003) consider a symmetric game with a continuum of atomistic small arbitrageurs. We are more interested in a richer market structure where both a large trader and a continuum of small traders are present. More specifically, we are interested in studying how the introduction of a large trader will change equilibrium outcomes. In our model, a large trader is defined by two characteristics: first, they have more precise information about the fundamental value of a currency. Second, a large trader can employ substantially larger amounts of wealth to launch a currency attack. The wealth that a large trader employs can come from their own capital, or more importantly, from their accessibility to credit due to their reputation. This is exactly how highly leveraged financial institutions finance their speculation.

Using our model, we find some interesting, and sometimes counter-intuitive, results. In the case in which the large trader has perfect information about the overvaluation of a currency, we find: first, his presence will accelerate the collapse of a fixed rate regime. Second, the collapse of the fixed rate regime is accelerated due to two reasons: (1) a large trader brings in additional speculative capital to an attack. (2) the presence of a large trader induces more aggressive strategies on the part of small traders. Third, the presence of a large trader will accelerate the collapse of a regime, even assuming that a large trader will not bring additional speculative capital to an attack, but will only change the *distribution* of the total speculative capital. (That is, with a large trader, the total speculative capital is more concentrated, instead of being evenly distributed over all traders.) Fourth, the larger the wealth of a large trader, the sooner the fixed exchange rate regime collapses. Fifth, we find that the presence of a large trader does not necessarily reduce the "bubble", which is defined as the duration between the time when the traders aware of the overvaluation have enough attacking capital to burst the bubble, and the time when the currency peg is actually broken.

In the case where the large trader has incomplete information, we find that: First, the presence of a large trader greatly increases the unpredictability of the time when the regime collapses, especially when the large trader has very imprecise information. This increases the difficulty for traders to time the collapse of the regime. We find this result especially interesting. In the model with a continuum of small traders, the time when the regime collapses is certain, given the time when the overvaluation starts is known. However, with the introduction of a large trader with incomplete information, we find that the time when the regime collapses now depends crucially on when the large trader becomes aware of the overvaluation, which could be any time within the large trader's awareness time window. Now even with perfect information about the time when the overvaluation starts, we can only predict the distribution about the time when the regime collapses. This greatly increases the uncertainty for traders to time the collapse of the regime. Our model demonstrates that the feature of perfect predictability of a crash, given the time when the overvaluation starts is known, is quite fragile. It depends critically on the assumption that all traders are negligibly small and identical. Thus, our model reveals that "timing a crash" and making profits from "riding a bubble" is difficult because of randomness.

Second, we find that the presence of the large trader may delay the collapse of the regime ex post, especially when the large trader has noisy information (but more precise information than small traders) and a large amount of wealth. We find that this result is interesting since it is different from the usual perception that the presence of a large trader will facilitate arbitrage and reduce asset mispricing. The intuition behind our result is as follows. There are two incentives driving a large trader to decide when to attack: the incentive to preempt other traders, and the incentive to "ride the overvaluation". When the large trader has incomplete information about the time when the overvaluation starts, his decision on when to attack crucially depends on when he becomes aware of the overvaluation. Conditional on becoming aware of the overvaluation *late* in his time window, his belief about the collapse of the regime will be delayed, and his incentive to "ride the overvaluation" will induce him to attack later. In this case, the collapse of the regime may be delayed ex post in the presence of the large trader. This delay can be severe when the large trader has a large amount of wealth, since the collapse of the regime now depends more on the large trader's action. This result is different from the case when the large trader has perfect information. In that case, we find that the large trader's incentive to preempt other traders dominates his incentive to "ride the overvaluation": the presence of the large trader always accelerates the collapse of the regime.

In sum, we find that the role of a large trader in a currency attack depends crucially on the quality of his information. A large trader with high quality information and a large amount of wealth can correct the overvaluation efficiently and accelerate the collapse of the regime. On the other hand, a large trader with low quality information and a large amount of wealth can severely strengthen the overvaluation and delay the collapse of the regime.

All of our results can be generalized to any asset markets. For example, the model can be easily interpreted as a study on foreign exchange markets without central bank intervention. Instead, we could assume that the bubble occurs because noise traders in the foreign exchange market believe that the current exchange rate level can be maintained and will trade at this level. In this way, our model can be used to explain the "currency bubbles and crashes" in foreign exchange markets with a floating exchange rate regime as mentioned in Brunnermeier, Nagel and Pedersen (2009). We can further generalize our results to any asset market with behavioral traders. In our model, we assume that the market price of a currency is unchanged, but its fundamental value is falling. More generally asset bubbles are often assumed to reflect rising asset prices, while its fundamental value stops growing. There is no essential difference in these two situations since what a arbitrageur cares about is the difference between the market price and its fundamental value, and not the direction of asset price changes. Thus, our results can be applied to any asset bubble and subsequent crash.

Our paper contributes to two strands of literature. The first strand of literature is on asset bubbles and crashes. The second strand of literature is on currency attacks. Our contribution to the first strand of literature is to introduce a large trader to Abreu and Brunnermeier (2003), and examine how the presence of a large trader will affect the evolvement of asset bubbles and subsequent crashes, in an environment where all traders are *rational* but become sequentially aware of a bubble. Our model also complements the currency attack literature. There are two strands of the currency attack literature closely related to our paper: One strand consists of currency attack models introducing incomplete information into the first generation currency attack models to explain the large devaluation observed after a fixed exchange rate regime collapses. The other strand consists of the currency attack models examining the role of a large trader in a currency attack. A detailed literature survey will be given in Section 2.

The rest of the paper consists of six sections. Section 2 provides a literature survey. Section 3 discusses the basic model and characterization of a dynamic currency attack. The model is a simplified version of the Abreu and Brunnermeier (2003) model: the model has a continuum of small arbitrageurs trading in a currency with a fixed exchange rate, that is open to a currency attack. We provide a characterization of the equilibrium and comparative statics. Section 4 introduces a large trader with perfect information, proves that there is a unique equilibrium, characterizes that equilibrium, and conducts comparative statics for the model. Section 5 introduces a large trader with incomplete information and examines its role. Section 6 concludes with observations on further possible extensions.

## 2 Literature Survey

Abreu and Brunnermeier (2003) construct a dynamic coordination game to explain the existence of asset bubbles, even in the presence of rational arbitrageurs who are capable of bursting the bubble. We have discussed their model above. Brunnermeier and Morgan (2008) change the assumption of a continuum of small traders in Abreu and Brunnermeier (2003), assuming a finite number of small traders. Nonetheless, their focus is still on *symmetric* equilibria. Since we focus on the study of the role that a large trader plays in a currency attack, our model exhibits a richer market structure where both a large trader and a continuum of atomistic traders co-exist. Moreover, equilibrium strategies between a large trader and small traders are *asymmetric* in our model. Our another modification to their model is that we use the uniform distribution, rather than the exponential distribution to characterize traders' belief. This assumption simplifies our analysis.

Following Abreu and Brunnermeier (2003), a large body of literature studies asset bubbles and crashes in a setup where traders have an incentive to "ride the bubble". Among them, Matsushima (2008) studies asset bubbles and crashes in a modified timing game. Instead of assuming sequential awareness of traders, he assumes that some traders are behavioral and always "ride the bubble". He demonstrates that even with a small probability that an arbitrageur is behavioral, rational arbitrageurs are willing to "ride the bubble", given that the information about whether an arbitrageur is rational or not is incomplete. Our model differs from his, because our focus is on how a *rational* large trader, will affect the evolvement of asset bubbles and crashes in a model where all traders are *rational*.

Both Rochon (2006) and Gara Minguez-Afonso (2007) apply Abreu and Brunnermeier (2003) to the first generation currency attack models to explain the devaluation that we observe when a fixed exchange rate regime collapses. The most important difference between our model and theirs is that our model focuses on the role of large traders in a currency attack with imperfect common knowledge, while they study currency attacks in a model without large traders. In addition, even in our basic model without large traders, the way in which we model a currency attack is also slightly different from theirs. We model the payoff structure of traders who try to gain from the devaluation, while they model the payoff structure of the attackers who try to avoid a capital loss associated with devaluation. They assume, as in Abreu and Brunnermeier (2003), exponential beliefs for traders.

Broner (2008) studies currency attacks in a first generation model of currency crises by introducing incomplete information about the threshold level of attacking pressure, or the foreign exchange reserves of a central bank. He assumes that some consumers have perfect information about the threshold level, while some do not. This assumption generates a sudden, discrete devaluation of a currency when the fixed exchange rate regime collapses. It is especially interesting to compare this paper to ours because the perfectly informed consumers are similar to a large trader with perfect information in our model. Our paper differs from his in the following ways: First, we introduce incomplete information by assuming sequential awareness of traders, following Abreu and Brunnermeier (2003). Second, we study currency attacks given a more general framework which can be applied to any asset market. Therefore, our results can be generalized to any asset markets. His model is set up in a more specific currency attack situation with certain assumptions about monetary policies. Third, we study a more general case when a large trader has both perfect information and incomplete information. His model focuses on the case when some traders have perfect information. We attain different results in the case when a large trader has perfect information. Broner (2008) finds that when the proportion of perfectly informed consumers is large enough, discrete devaluation will not happen. While in our model with a single large trader with perfect information, even if total speculative capital is large enough, the regime will not collapse immediately when the overvaluation starts. The major reason that we have a different result is that Broner (2008) studies symmetric strategies by *a continuum of* consumers with perfect information. While in our model we are studying *a single* large trader acting more like a monopolist. The market power of a single large trader gives him a greater incentive to "ride the overvaluation", and induces him to delay the attack, even when he has perfect information. We reach the same conclusion that the presence of a large trader with perfect information will accelerate the collapse of the regime. In our more general case where a large trader has more precise (but not perfect) information than small traders, we find that the presence of a large trader can actually delay the collapse of the regime, especially when a large trader has noisy information and a large amount of speculative capital.

Morris and Shin (1998) study currency attacks in a one-period global game setup. They demonstrate that, although a self-fulfilling currency attack game has multiple equilibria when economic fundamentals are common knowledge, it has a unique equilibrium when traders can only observe the fundamentals with small noise. Successfully overcoming the problem of indeterminacy of multiple equilibria models, their model allows the analysis of policy implications. Corsetti, Dasgupta, Morris and Shin (2004) extend the Morris and Shin (1998) model to one with a large trader. They analyze two cases where the large trader has, and has not, a signalling function. They find that in both cases the presence of a large trader does increase the possibility of the collapse of a fixed exchange rate regime, and make small traders more aggressive.

Correstti, Pesenti, and Roubini (2001) give a comprehensive survey on the role that large traders play in currency attacks. In the theoretical section of their survey, they apply a traditional coordination game with perfect information, and then a global game established by Corsetti, Dasgupta, Morris and Shin (2004) to the study of the role of a large trader in a currency market. In the empirical section, they combine both econometric analysis and case studies to explore examples of currency attacks. Their conclusion is that both theoretical and empirical studies reveal that large traders do have a significant role in currency attacks, and more academic research is required to address a number of issues, including the dynamics of currency attacks or crises.

Bannier (2005) modifies the model established by Corsetti, Dasgupta, Morris and Shin (2004) by changing the assumption about a central bank's strategy. Due to that modification, both the large trader and small traders' strategies are symmetric and analytical results are available. She finds that this modification changes the results given by Corsetti, Dasgupta, Morris and Shin (2004). Now a large trader can increase the possibility of a regime collapse only when market sentiment is pessimistic. However, the presence of a large trader will decrease the possibility of a regime collapse when the market sentiment is optimistic.

Due to the features of our model, our study of a large trader in currency attacks focuses on a different aspect compared to these two papers. They study the possibility of the collapse of a fixed exchange rate regime in a static model, and whether the presence of large traders will increase or decrease that possibility. In our model, currency devaluation is inevitable, and the issue that we focus on is *when* it will happen. Thus our study focuses on whether the presence of a large trader will *accelerate* or *delay* a currency attack. Here we do not give a formal welfare analysis to examine whether the presence of a large trader is beneficial or harmful to an economy. However, in general, we believe that a currency overvaluation is harmful to an economy, and early correction is always better than a late one if the correction is inevitable. In this sense, a late collapse of the regime will do more harm to an economy than an early one.

# 3 The Benchmark Model with Small Traders and No Large Trader

## 3.1 Environment

In this model, we apply a simplified version of the Abreu and Brunnermeier (2003) model to the foreign exchange market. We capture the essence of their idea that the difficulty in coordination among arbitrageurs, together with their incentive to time the market, can cause asset mispricing. Instead of adopting the exponential distribution in their setup, we use the uniform distribution to simplify the calculation.

Assume that there is a country with a fixed exchange rate regime where a central bank commits to maintaining the exchange rate at a fixed level until it exhausts all of its foreign reserves, whose level is denoted by $k > 0$.

From time $t_0$, the exchange rate becomes overvalued relative to its fundamental value, at a rate of $g$. Denote the initial exchange rate as $E_0$. The fundamental exchange rate at $t$ is $E_0$ when $t < t_0$ and $E_0(1 + g(t - t_0))$ when $t \geq t_0$. Here the exchange rate is denominated in the domestic currency, say wons. So $E_0$ means that 1 dollar can exchange for $E_0$ wons.

Without any currency attacks, the fixed exchange rate regime will collapse at some exogenously given time $t_0 + \tau'$. This assumption captures the idea that any asset mispricing is not sustainable in the long run. We follow Abreu and Brunnermeier (2003) in making this simplified assumption to avoid ever greater currency overvaluations. Figure 1 shows how the fundamental exchange rate changes with time.

There is a continuum of traders of mass 1. Each trader is financially constrained

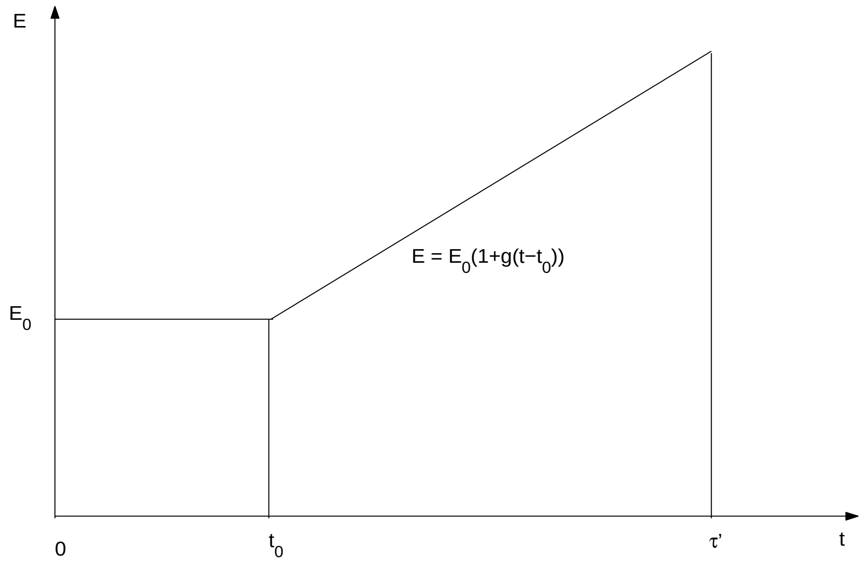


Figure 1: How the fundamental exchange rate $E$ changes with time $t$

and can only access the credit whose worth is normalized to 1 dollar. Each trader has to choose from two strategies: attacking or refraining. When $t < t_0 + \tau'$, the exchange rate will devalue to the fundamental value if and only if attacking pressure exceeds $k$. This assumption follows that of Obstfeld (1996) and Morris and Shin (1998) and captures the idea of market liquidity.

We specify the payoff structure of traders as follows: if they choose refraining, which means that they will do nothing, they will gain zero. If they choose to attack, they will borrow wons from the banks of the attacked country, then exchange them into dollars from the central bank. The costs of attacking consist of two parts. One part is the fixed transaction costs associated with the currency exchanges, which is denoted by $c^F$. We assume that the fixed transaction costs are not so high that they prevent the traders from ever attacking, despite the awareness of the overvaluation. The other part is the interest differential between wons and dollars, since we assume that the interest rate of wons is higher than that of dollars. Let $c$ denote the interest differential. Thus, if a trader keeps attacking during a time interval $\triangle t$, he will incur the cost of $c.\triangle t$. The payoffs of traders from attacking is as follows. If the regime collapses at instant $t$, the payoffs of a trader attacking at instant $t$ with the wealth of 1 dollar will depend on how many other traders are attacking. If the attacking mass is less than or equal to $k$, his payoffs are $E_0.g(t-t_0)$. If the attacking mass is greater than $k$, only the first randomly chosen mass $k$ of attacking traders will gain the payoffs of $E_0.g(t-t_0)$. So given the attacking pressure $\alpha > k$, the expected payoffs of a trader are given by $\frac{k}{\alpha}E_0.g(t-t_0)$. For simplicity of the analysis, we assume that no partial attacking is allowed.

The traders only have incomplete information about $t_0$, the time at which the overvaluation begins. More specifically, all the traders have a prior belief about $t_0$, which is denoted by $\Phi(t_0)$. We assume that the traders have an improper uniform belief about $t_0$ over $[0, \infty)$.

From $t_0$, a new cohort of small traders with mass $\frac{1}{\eta}$ becomes aware of the overvaluation in each instant from $t_0$ until $t_0+\eta$. So $\eta$ is the time window for all the traders to become aware of the overvaluation.

Suppose that a trader becomes aware of the overvaluation at time $t_i$. We also denote this trader by $t_i$. Conditional on $t_i$, trader $t_i$'s belief about $t_0$ is given by the CDF

$$\Phi(t_0|t_i) = \frac{t - t_i + \eta}{\eta}, \tag{1}$$

where $t \in [t_i - \eta, t_i]$.

Given such a setup, we try to find the equilibrium strategy of a rational trader $t_i$.

Let $\sigma(t, t_i)$ denote the strategy of trader $t_i$ and the function $\sigma : [0, \infty) \times [0, \infty) \mapsto \{0, 1\}$ a strategy profile. We assume that traders will act only after being informed. Thus, trader $t_i$'s strategy is given by $\sigma(., t_i) : [t_i, t_i + \tau'] \mapsto \{0, 1\}$, where 0 means refraining and 1 means attacking. The aggregate attacking pressure of all the traders at time $t \geq t_0$ is given by

$$s(t, t_0) = \int_{t_0}^{min\{t, t_0+\eta\}} \sigma(t, t_i) dt_i. \tag{2}$$

Let

$$T^*(t_0) = inf\{t | s(t, t_0) \geq k \text{ or } t = t_0 + \tau'\} \tag{3}$$

denote the collapse time of the fixed exchange rate regime for a given realization of $t_0$. Recall that $\Phi(.|t_i)$ denotes trader $i$'s belief about $t_0$ given that $t_0 \in [t_i - \eta, t_i]$. Hence, his belief about the collapse time is given by the CDF

$$\Pi(t|t_i) = Prob(T^*(t_0) < t|t_i)$$

So $\Pi(t|t_i)$ gives us trader $t_i$'s belief on the probability with which the regime collapses before time $t$.

The time $t_i$ expected payoffs of trader $t_i$, who remains refraining until he begins to attack at time $t$ and keeps attacking afterward until the regime collapses, are given by

$$\int_t^{t_i+\tau'} E_0[g(s - T^{*-1}(s)) - c(s - t)] d\Pi(s|t_i) - c^F,$$

provided that the attacking pressure at $t$ does not strictly exceed $k$ and that $T^*(.)$ is strictly increasing in $t_0$. Later we will show that in equilibrium all the conditions will hold.

If we normalize the initial exchange rate $E_0$ to 1, we get:

$$\int_{t}^{t_i+\tau'} [g(s - T^{*-1}(s)) - c(s - t)] d\Pi(s|t_i) - c^F. \tag{4}$$

## 3.2 Equilibrium Characterization

We confine our attention to symmetric trigger strategies. We can prove that there is a unique symmetric trigger strategy equilibrium. In this equilibrium, each trader $t_i$ will attack at the instant $t_i + \tau^*$ and keep attacking until the regime collapses. (Rochon (2006) proves in a similar setup that this symmetric trigger strategy equilibrium is a strongly rational expectation equilibrium in the set of strategies, with the only restriction that traders act after being informed). Depending upon parameter values of $\eta$, $k$, $g$ and $c$, the regime can collapse exogenously or endogenously. Here we will focus on the endogenous collapse case.

**Proposition 1.** *Given $\tau' > \frac{c}{g}k\eta$ and $c \geq g$, there is a unique symmetric trigger strategy equilibrium where the regime collapses endogenously. In this equilibrium, each trader $t_i$ begins to attack at the instant $t_i + \tau^*$ and keeps attacking until the regime collapses, where $\tau^* = \frac{c-g}{g}k\eta$. In equilibrium the regime collapses exactly at the instant $t_0 + k\eta + \tau^*$.*

*Given $\tau' > k\eta$ and $c < g$, there is a unique symmetric trigger strategy equilibrium where the regime collapses endogenously. In this equilibrium, each trader $t_i$ begins to attack at the instant $t_i$ and keeps attacking until the regime collapses. In equilibrium the regime collapses exactly at the instant $t_0 + k\eta$.*

**Proof:**

Let $\tau^*$ define a symmetric trigger equilibrium. That is, all the traders begin to attack at $t_i + \tau^*$. Given such a strategy, the regime will collapse when trader $t_0 + k\eta$ attacks, and the collapsing time will be $t_0 + k\eta + \tau^*$.

Now consider the optimal strategy of trader $t_i$ given that all the other traders take the strategy $\tau^*$. Thus the regime will collapse at $t_0 + \zeta$, where $\zeta = k\eta + \tau^*$. Trader $t_i$ believes that $t_0 \in [t_i - \eta, t_i]$, the CDF of his posterior belief about $t_0$ is given by

$$\Phi(t|t_i) = \frac{t - t_i + \eta}{\eta}. \tag{5}$$

Since the collapsing time is $t_0 + \zeta$, he believes that $t_0 + \zeta \in [t_i - \eta + \zeta, t_i + \zeta]$. The CDF of his posterior belief about the collapsing date $t_0 + \zeta$ at time $t_i + \tau$ is given by

$$\Pi(t_i + \tau|t_i) = \frac{t_i + \tau - (t_i - \eta + \zeta)}{\eta} = \frac{\tau + \eta - \zeta}{\eta}. \tag{6}$$

Trader $t_i$'s expected payoff from attacking at $t$ and keeping attacking until the regime collapses is given by:

$$\int_t^{t_i+\zeta} (g(s - T^{*-1}(s)) - c(s - t))d\Pi(s|t_i) - c^F. \tag{7}$$

The first order condition gives the optimal $\tau$ for him to attack:

$$\frac{\pi(t_i + \tau|t_i)}{1 - \Pi(t_i + \tau|t_i)} = \frac{c}{g(t_i + \tau - T^{*-1}(t_i + \tau))}. \tag{8}$$

We also check the second order condition, which turns out that the second order derivative is negative and the second order condition is satisfied.

Taking Equation (6) into the left hand side of the first order condition gives us:

$$\frac{\pi(t_i + \tau|t_i)}{1 - \Pi(t_i + \tau|t_i)} = \frac{1}{\zeta - \tau}. \tag{9}$$

In addition, in this symmetric equilibrium, the duration between the time when the regime collapses and the time when the overvaluation happens is given by: $t_i +$

$\tau - T^{*-1}(t_i + \tau) = \tau^* + k\eta = \zeta$. This is because each trader will delay a period of $\tau^*$ and the regime will collapse exactly at the moment $t_0 + k\eta + \tau^*$ when the trader $t_0 + k\eta$ launches his attack.

So we find:

$$\frac{1}{\tau^* + k\eta - \tau} = \frac{c}{g(\tau^* + k\eta)}. \tag{10}$$

Since it is a symmetric equilibrium, $\tau = \tau^*$. Solving the above equation, we get

$$\tau^* = \frac{(c-g)k\eta}{g}. \tag{11}$$

Given $\frac{c}{g}k\eta < \tau'$, the regime will collapse at $t_0 + k\eta + \tau^* < t_0 + \tau'$ endogenously.

Notice that $\tau^* \geq 0$ if and only if $c \geq g$. When $c < g$, we will get the corner solution of $\tau^* = 0$.

**Q.E.D**

The intuition of the equilibrium is as follows. Given that all the traders begin their attack at $t_i + \tau^*$, the instantaneous probability that the regime collapses at $t_i + \tau$ of trader $t_i$ is given by:

$$\frac{\pi(t_i + \tau | t_i)}{1 - \Pi(t_i + \tau | t_i)} = \frac{1}{\tau^* + k\eta - \tau}. \tag{12}$$

If the regime exactly collapses at $t_i + \tau$, the gains from attacking will be $g(\tau^* + k\eta)$. Thus, the expected marginal benefits of trader $t_i$ attacking at $t_i + \tau$ are given by:

$$g(\tau^* + k\eta)\frac{\pi(t_i + \tau | t_i)}{1 - \Pi(t_i + \tau | t_i)} = g(\tau^* + k\eta)\frac{1}{\tau^* + k\eta - \tau}.$$

Meanwhile, the marginal costs incurred by attacking at time $t_i + \tau$ are $c$, which are constant. From the above equations we can see that the expected marginal gains from attacking are strictly increasing in $\tau$, since the trader $t_i$'s subjective instantaneous probability that the regime collapses at time $t_i + \tau$ is strictly increasing in $\tau$. So there is a unique level of $\tau$, where the expected marginal gains from attacking are exactly equal to the marginal costs incurred by attacking. And it is the optimal time for trader $t_i$ to attack. Figures 2 and 3 explain the intuition.

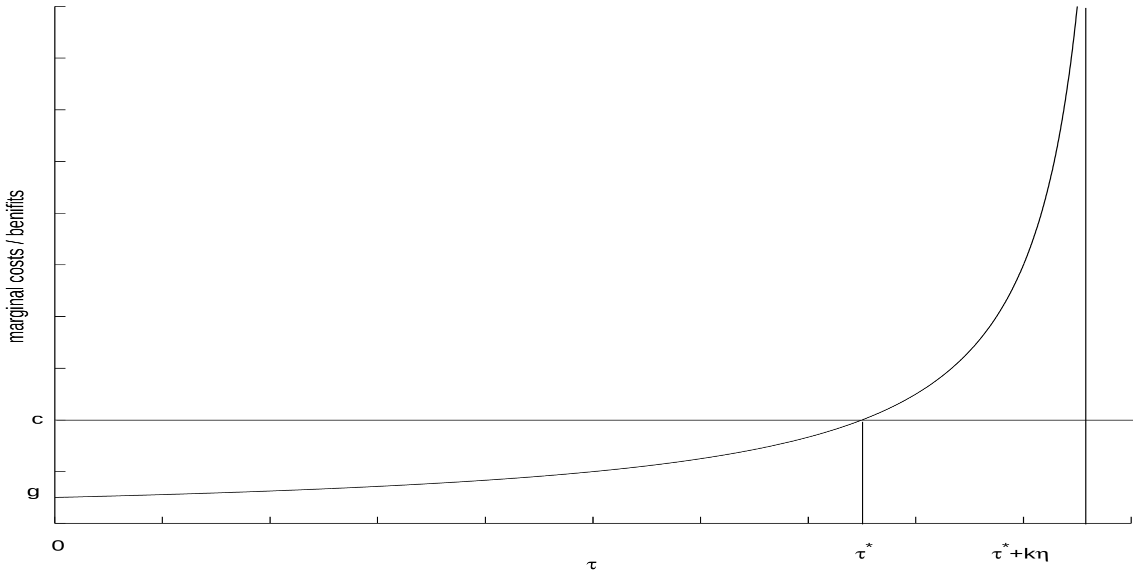


Figure 2: How the marginal costs and benefits change in $\tau$ in the case of the interior solution of $\tau^*$

## 3.3 Comparative Statics

This section studies how the changes in parameters of the model influence equilibrium results.

We know that in equilibrium

$$\tau^* = \frac{(c-g)k\eta}{g}.$$

First, we can see that the traders will wait longer with higher $c$. The intuition is simple. Higher $c$ means that it will cost more if a trader launches an attack early. Hence a trader would like to wait longer to reduce the costs of attacking.

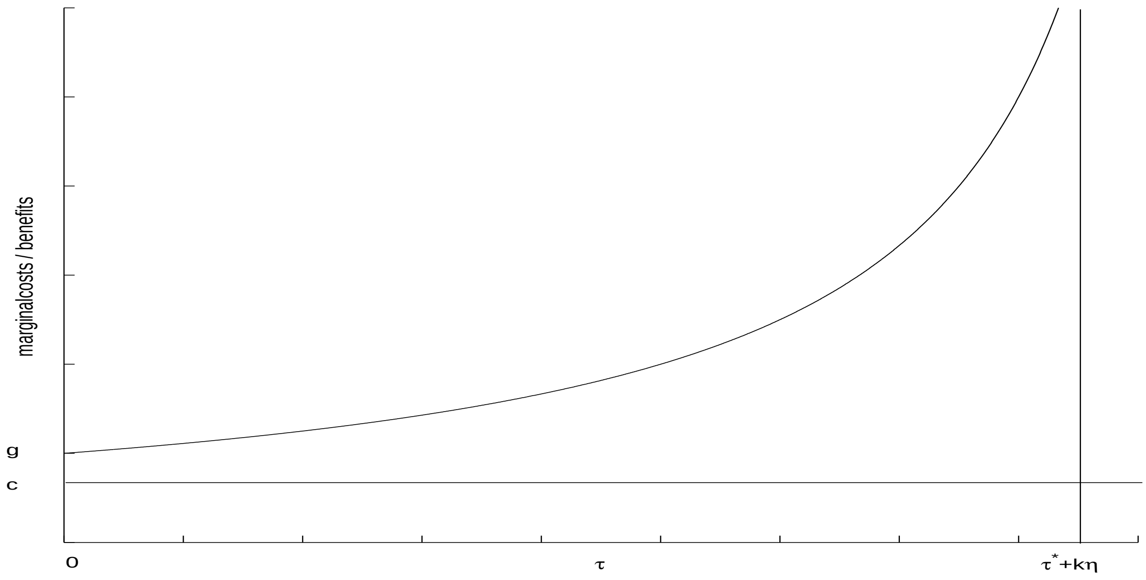


Figure 3: How the marginal costs and benefits change in $\tau$ in the case of the corner solution of $\tau^*$

Second, we find that the traders will wait longer with both higher $k$ and $\eta$. This result is also intuitive. Higher $\eta$ means more dispersed opinions among the traders and higher $k$ means a higher requirement for coordination. Both will increase the difficulties in coordination and induce the traders to wait longer.

We know that $c$, $k$ and $\eta$ are all parameters indicating how difficult it is to arbitrage in a foreign exchange market. We find that now the frictions in the market become a blessing for the traders, since more frictions will induce the traders to wait longer and make higher profits from the overvaluation.

Third, we find that the traders will wait longer with lower $g$, the rate at which the currency is overvalued. In this case, higher $g$ increases the traders' incentive to preempt other traders and makes the traders less patient. In the extreme case when $g > c$, traders will launch an attack immediately after they become aware of the overvaluation.

Finally, there is an interesting result about the exchange rate level when the regime collapses, which determines the magnitude of the devaluation. It is given by $ck\eta$. We can see that $g$ does not play a role in determining the magnitude of the devaluation. This is because the speed at which the fundamental value of the currency decreases has two opposite effects: First, it affects the optimal delay time of traders. Second, it affects the fundamental exchange rate at time $t$. The net result from these two effects is that $g$ will not influence the exchange rate when the regime collapses.

# 4 The Model in which the Large Trader Has Perfect Information

In this section, we introduce a large trader into the basic model. We start with a simple model in which the large trader has perfect information about $t_0$, the time at which the overvaluation happens. Later in Section 5, we will generalize the model into the one in which the large trader has incomplete information about $t_0$.

Moreover, we assume that traders consist of one large trader with wealth $\lambda < k$ and a continuum of small traders of mass 1 with total wealth of 1. Here we assume $\lambda < k$ such that the large trader cannot independently break the peg. This assumption is realistic because even a large trader like Soros in financial markets cannot single-handedly break a currency peg. Last, we assume that the action of the large trader will not be observed by other traders. We also assume $c^F$ for simplicity since it does not affect equilibrium outcomes. All the other assumptions for small traders are unchanged.

## 4.1 Equilibrium Characterization

Proposition 2 characterizes the unique trigger strategy equilibrium in this game.

**Proposition 2.** *Given $c > g$, $\lambda > \frac{c}{c+g}k$, and $\tau' > \frac{c}{c+g}k\eta$, there is a unique Bayesian equilibrium where the regime collapses endogenously. In this equilibrium, each small trader begins to attack at the instant $t_i + \tau^{ST}$ and keeps attacking until the regime collapses. Here $\tau^{ST} = \frac{c-g}{c+g}k\eta$. The large trader begins to attack at $t_0 + \tau^{LT}$, where $\tau^{LT} = \frac{c}{c+g}k\eta$. The regime collapses exactly at the time when the large trader launches the attack.*

*Given $c > g$, $\lambda \leq \frac{c}{c+g}k$, and $\tau' > \frac{c}{g}(k-\lambda)\eta$, there is a unique Bayesian equilibrium where the regime collapses endogenously. In this equilibrium, each small trader begins to attack at the instant $t_i + \tau^{ST}$ and keeps attacking until the regime collapses. Here $\tau^{ST} = \frac{c-g}{g}(k-\lambda)\eta$. The large trader begins to attack at $t_0+\tau^{LT}$, where $\tau^{LT} = \frac{c}{g}(k-\lambda)\eta$. The regime collapses exactly at the time when the large trader launches the attack.*

*Given $c \leq g$, $\lambda > \frac{1}{2}k$, and $\tau' > \frac{1}{2}k\eta$, there is a unique Bayesian equilibrium where the regime collapses endogenously. In this equilibrium, each small trader begins to attack at the instant $t_i$ and keeps attacking until the regime collapses. The large trader begins to attack at $t_0 + \tau^{LT}$, where $\tau^{LT} = \frac{1}{2}k\eta$. The regime collapses exactly at the time when the large trader launches the attack.*

*Given $c \leq g$, $\lambda \leq \frac{1}{2}k$, and $\tau' > (k-\lambda)\eta$, there is a unique Bayesian equilibrium where the regime collapses endogenously. In this equilibrium, each small trader begins to attack at the instant $t_i$ and keeps attacking until the regime collapses. The large trader begins to attack at $t_0+\tau^{LT}$, where $\tau^{LT} = (k-\lambda)\eta$. The regime collapses exactly at the time when the large trader launches the attack.*

**Proof:**

Since the large trader has perfect information about $t_0$, he will choose the optimal time $t_0 + \tau^{LT}$ to maximize his profits, given the equilibrium strategies taken by small traders. Since small traders are identical ex ante and atomically small, they will take symmetric strategies. Suppose that each small trader plays the symmetric trading strategy $t_i + \tau^{ST}$ in equilibrium. From the moment of $t_0 + (k-\lambda)\eta + \tau^{ST}$ on, the total wealth of the large trader and small traders exceeds the threshold level $k$. Thus, the payoffs of the large trader from attacking at $t_0 + (k-\lambda)\eta + \tau^{ST} + t$ are given by

$$g[(k-\lambda)\eta + \tau^{ST} + t](\lambda - \frac{t}{\eta}),$$

where $0 \leq t \leq \lambda\eta$.

Notice that $t \leq \lambda\eta$, or the regime will collapse solely due to the attacking pressure from small traders, and the large trader will gain zero profits. The large trader will choose an optimal level of $t$ to maximize his expected profits. Solving the maximization problem, we get that $t^* = max\{(\lambda - \frac{1}{2}k)\eta - \frac{\tau^{ST}}{2}, 0\}$.

Now let us look at the best responses of small traders. Our previous proof for the unique symmetric trigger strategy equilibrium still holds in this case. Only now the optimal attacking time $t_i + \tau^{ST}$ is determined by the following conditions.

Given $(\lambda - \frac{1}{2}k)\eta - \frac{\tau^{ST}}{2} > 0$, the optimal strategy of the large trader is $t^* = (\lambda - \frac{1}{2}k)\eta - \frac{\tau^{ST}}{2}$. Thus, in equilibrium the regime collapses at $T^* = t_0 + \zeta = t_0 + \frac{1}{2}(k\eta + \tau^{ST})$. Therefore, the first order condition gives

$$\frac{\pi(t_i + \tau^{ST}|t_i)}{1 - \Pi(t_i + \tau^{ST}|t_i)} = \frac{1}{\zeta - \tau^{ST}} = \frac{c}{g\zeta}.$$

In equilibrium, $\zeta = \frac{1}{2}(k + \tau^{ST})$. Thus we get $\tau^{ST} = max\{\frac{c-g}{c+g}k\eta, 0\}$. Given $c > g$, the large trader's equilibrium strategy is $\tau^{LT} = (k-\lambda)\eta + \tau^{ST} + (\lambda - \frac{1}{2}k)\eta -$

$\frac{\tau^{ST}}{2} = \frac{c}{c+g}k\eta$. Checking the condition inducing the large trader to choose positive $t^*$, $(\lambda - \frac{1}{2}k)\eta - \frac{\tau^{ST}}{2} > 0$, we get:

$$\lambda > \frac{c}{c+g}k.$$

Therefore, we get the first equilibrium in Proposition 2.

Similarly, we can find the second, third and fourth equilibria depending on different parameter values of $c$, $g$, $\lambda$ and $k$.

**Q.E.D.**

## 4.2 The Role of the Large Trader

In this section, we analyze the role that a large trader plays in a currency attack based on our model. Our findings are the following:

1. In general, the presence of a large trader accelerates the collapse of the regime.

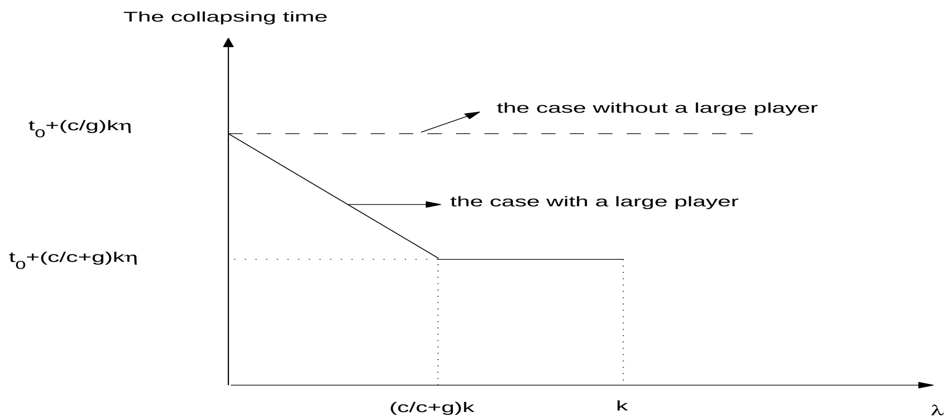


Figure 4: Collapsing time of the regime when $c > g$

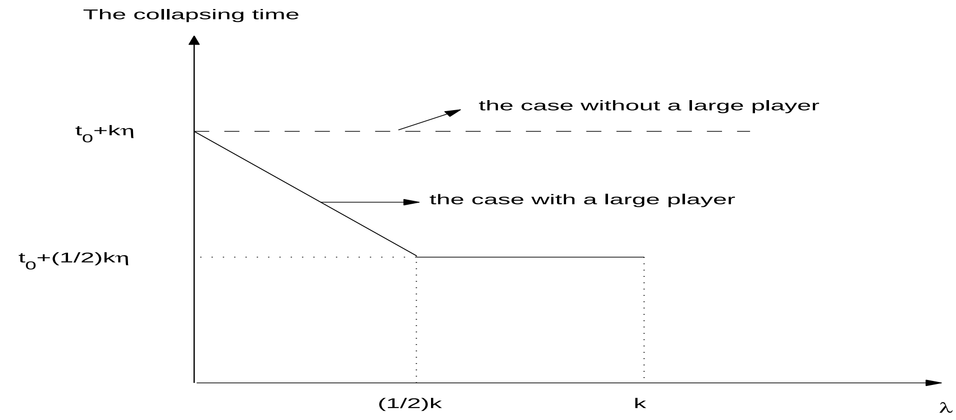


Figure 5: Collapsing time of the regime when $c \leq g$

Figure 4 and 5 illustrate how the introduction of a large trader changes the collapsing time of the regime. We can tell that in both cases of $c > g$ and $c \leq g$, the introduction of a large trader accelerates the collapse of the regime.

2. The collapse of the regime is accelerated due to two reasons: first, additional wealth of $\lambda$ is available from $t_0$ for the attack with the introduction of a large trader. Second, the presence of a large trader makes small traders more aggressive and shorten their waiting time $\tau^{ST}$.

   The first reason is straightforward to understand. The second reason can be explained by comparing $\tau^{ST}$ in the case with and without the large trader. Recall that in the case without a large trader, $\tau^{ST} = \frac{(c-g)k\eta}{g}$ when $c > g$, and $\tau^{ST} = 0$ when $c \leq g$. In the case with a large trader, given $c > g$ and $\lambda > \frac{c}{c+g}k$, $\tau^{ST} = \frac{c-g}{c+g}k\eta < \frac{(c-g)k\eta}{g}$. Given $c > g$ and $\lambda \leq \frac{c}{c+g}k$, $\tau^{ST} = \frac{c-g}{g}(k-\lambda)\eta < \frac{(c-g)k\eta}{g}$. Thus we can see in the case $c > g$, small traders will shorten their waiting time in equilibrium. In the case of $c \leq g$, $\tau^{ST} = 0$ in both cases with and without the presence of a large trader.

3. So far we have studied the role of a large trader by assuming that the presence of a large trader will bring some extra wealth for attacking. Without a large trader, the total potential attacking wealth is 1. With a large trader, the total potential attacking wealth is $1 + \lambda$ with $\lambda$ of which aware of the overvaluation at $t_0$, and 1 of which aware of the overvaluation over the time window $t_0 + \eta$.

   Now we are interested in studying what will happen if the presence of a large trader does not change the *total amount* of potential attacking wealth. Instead, we assume that the presence of a large trader only changes the *distribution* of the attacking wealth. More specifically, we set up a benchmark case in which the total attacking wealth of $1 + \lambda$ is evenly distributed over all small traders who become aware of the overvaluation over $[t_0, t_0 + \eta]$. Then we compare this benchmark case with our model with a large trader. We find that the presence of a large trader will accelerate the collapse of the regime compared to the benchmark case.

   In the benchmark case without a large trader, the regime will collapse at $t_0 + \frac{k}{1+\lambda}\eta + \tau^*$, where $\tau^* = \frac{c-g}{g}\frac{k}{1+\lambda}\eta$. Thus, the regime collapses at $t_0 + \frac{c}{g}\frac{k}{1+\lambda}\eta$ (here we only consider the case of $c > g$). Recall that in the case with a large trader, the regime collapses at $t_0 + \frac{c}{c+g}k\eta$ given $\lambda > \frac{c}{c+g}k$, and at $t_0 + \frac{c}{g}(k - \lambda)\eta$ given $\lambda \leq \frac{c}{c+g}k$.

   Comparing these two cases, we find that the presence of a large trader will definitely accelerate the collapse of the regime (see Appendix A for a detailed derivation). Later we will study the case when the large trader has incomplete information about the time when the currency is overvalued. We find that this result may not hold in this case.

4. In general, the larger the wealth of a large trader is, the sooner the regime collapses. That is, $\tau^{LT}$ is non-increasing in $\lambda$. Our model reveals an interesting relationship between the wealth of a large trader and the collapsing time of the regime. When $\lambda$ is small enough, there is a strictly decreasing relationship between these two variables. However, when $\lambda$ is large enough, the collapsing time will be invarient to changes in $\lambda$. This relationship is true in both cases of $c > g$ and $c \leq g$. The only difference is that in the first case of $c > g$, the critical value for $\lambda$ is $\frac{c}{c+g}k$. While in the case of $c \leq g$, the critical value for $\lambda$ is $\frac{1}{2}k$.

5. The presence of a large trader does not necessarily reduce the"bubble", which is defined as the time interval between the moment when traders with total wealth of $k$ become aware of the overvaluation and the moment when the regime actually collapses. Given the definition of the "bubble" in our model, in the case without a large trader, the "bubble" is $\frac{c-g}{g}k\eta$ when $c > g$. It is 0 when $c \leq g$. In the case with a large trader,

   (a) When $c > g$ and $\lambda > \frac{c}{c+g}k$, the "bubble" is $\frac{c}{c+g}k\eta - (k-\lambda)\eta < \frac{c}{c+g}k\eta$.

   (b) When $c > g$ and $\lambda \leq \frac{c}{c+g}k$, the "bubble" is $\frac{c-g}{g}(k-\lambda)\eta < \frac{c-g}{g}k\eta$ .

   (c) When $c \leq g$ and $\lambda > \frac{1}{2}k$, the "bubble" is $\frac{1}{2}k\eta - (k-\lambda)\eta > 0$.

   (d) When $c \leq g$ and $\lambda \leq \frac{1}{2}k$, the "bubble" is 0.

   Thus we find that:

   (a) In the case of $c \leq g$, the presence of a large trader will lead to a positive bubble when $\lambda > \frac{1}{2}k$. While there is no bubble at all when $c \leq g$ without a large trader. In this case, the presence of a large trader increases the "bubble".

(b) In the case when $c > g$ and $\lambda > \frac{c}{c+g}k$, the bubble could be increased or decreased, depending on parameter values of $c$, $g$ and $\lambda$. In addition, the bubble is strictly increasing in $\lambda$ in this case.

(c) When $c > g$ and $\lambda \leq \frac{c}{c+g}k$, the presence of a large trader definitely increases the "bubble".

# 5 The Model in Which the Large Trader Has Incomplete Information

Now let us examine a more general case in which the large trader has incomplete information about $t_0$. We are interested in this case because in reality even large traders cannot be absolutely certain about the precise moment at which the overvaluation starts. Moreover, with the incomplete information assumption, we can study how the degree of precision of the large trader's information influences the equilibrium outcomes. Thus, we can gain more insights about the role of large traders in a currency attack.

More specifically, we assume that $\eta^{LT}$ is the time window for the large trader to become aware of the currency overvaluation. That is, the probability that the large trader becomes aware of the overvaluation is evenly distributed over $[t_0, t_0 + \eta^{LT}]$. We use $t^{LT}$ to denote the moment at which the large trader becomes aware of the overvaluation.

The major results that we find in this section are:

1. With some reasonable parameter values, there is a unique trigger strategy perfect Bayesian equilibrium in the game. In this equilibrium, the large trader will launch the attack some certain periods, $\tau^{LT}$, after he becomes aware of the overvaluation and will keep attacking afterward until the regime collapses. All small traders will launch the attack some certain periods, $\tau^{ST}$, after he becomes aware of the overvaluation and will keep attacking afterward until the regime collapses.

2. The equilibrium strategies of both the large trader and small traders, $\tau^{LT}$ and $\tau^{ST}$, are increasing in the wealth of the large trader, $\lambda$.

3. When the precision of the large trader's information increases ($\eta^{LT}$ decreases), the equilibrium strategy of the large trader, $\tau^{LT}$ will be lower, and the equilibrium strategy of small traders, $\tau^{ST}$ will be higher.

4. The presence of a large trader greatly increases the unpredictability of the time when the regime collapses given $t_0$, the time when the overvaluation starts.

5. The presence of the large trader may delay the collapse of the regime ex post, especially when the large trader has very imprecise information and a large amount of wealth.

## 5.1 Equilibrium Characterization

First, we define a trigger strategy perfect Bayesian equilibrium in which small traders take a symmetric trigger strategy $t_i+\tau^{ST}$, and the large trader takes a trigger strategy

$t^{LT} + \tau^{LT}$ as follows:

1. For the large trader, given $\tau^{ST}$, $\tau^{LT}$ maximizes his expected profits.

2. For a small trader, given $\tau^{LT}$ played by the large trader and $\tau^{ST}$ played by all the other small traders, $\tau^{ST}$ maximizes his expected profits.

3. Each trader updates his belief whenever Bayes' rule is applied.

Now we need to specify the expected profit functions for both the large trader and small traders. For a large trader, his expected profit of attacking at $t$ given small traders' strategy $t_i + \tau^{ST}$ is as follows:

$$E(\Pi^{LT}|t^{LT}) = \int_{t^{LT}-\eta^{LT}+(k-\lambda)\eta+\tau^{ST}}^{t^{LT}+(k-\lambda)\eta+\tau^{ST}} f(t,s)d\Pi(s|t^{LT}), \tag{13}$$

where $\Pi(s|t^{LT})$ denotes the CDF of the large trader's belief about $t_0 + (k-\lambda)\eta + \tau^{ST}$, which is uniformly distributed over $[t^{LT} - \eta^{LT} + (k-\lambda)\eta + \tau^{ST}, t^{LT} + (k-\lambda)\eta + \tau^{ST}]$. In addition, we have

$$f(t,s) = \begin{cases} 0, & \text{when } t > s + \lambda\eta \\ (\lambda - \frac{t-s}{\eta})[(k-\lambda)\eta + \tau^{ST} + t - s]g, & \text{when } s < t \leq s + \lambda\eta \\ \lambda\{[(k-\lambda)\eta + \tau^{ST}]g - c(s-t)\}, & \text{when } t \leq s \end{cases}$$

The reason why we have the above expression is as follows:

1. When $t > s + \lambda\eta$, the regime already collapses when the large trader attacks, and the large trader's profit is 0.

2. When $s < t \leq s + \lambda\eta$, the large trader attacks in the range of $[t_0 + (k-\lambda)\eta + \tau^{ST}, t_0 + k\eta + \tau^{ST}]$, and the regime will collapse exactly at the moment when the

large trader attacks. Thus the large trader's profit is given by: $(\lambda - \frac{t-s}{\eta})[(k - \lambda)\eta + \tau^{ST} + t - s]g$.

3. When $t \leq s$, the large trader attacks before $t_0 + (k-\lambda)\eta + \tau^{ST}$, and the regime will collapse exactly at $t_0 + (k - \lambda)\eta + \tau^{ST}$. Thus the large trader's profit is given by: $\lambda\{[(k-\lambda)\eta + \tau^{ST}]g - c(s-t)\}$.

The large trader will choose the optimal attacking time $t$ to maximize his profits. By solving the large trader's profit maximization problem, we will find the large trader's best response of $\tau^{LT}$ as a function of $\tau^{ST}$.

Due to the property of the large trader's profit function, it is extremely difficult to find the analytical solution for the large trader's best response function. Numerical solution will be given in Section 5.2.

For a small trader, his expected payoff of attacking at $t$ given the large trader's strategy of $\tau^{LT}$ and all the other small traders taking the strategy $\tau^{ST}$ crucially depends on different combinations of $\tau^{LT}$, $\tau^{ST}$, $(k-\lambda)\eta$, $k\eta$ and $\eta^{LT}$. It turns out that there are six different combinations and the small trader's expected profit is as follows under each combination:

1. When $t_0 + \tau^{LT} \geq t_0 + k\eta + \tau^{ST}$, the regime collapses at $t_0 + k\eta + \tau^{ST}$.

   (a) If $t > t_i + k\eta + \tau^{ST}$, $E(\Pi^{ST}|t_i) = 0$ since the regime already collapses when the small trader attacks.

   (b) If $t_i - \eta + k\eta + \tau^{ST} \leq t \leq t_i + k\eta + \tau^{ST}$,

$$E(\Pi^{ST}|t_i) = \int_t^{t_i + k\eta + \tau^{ST}} \{g[k\eta + \tau^{ST}] - c(s-t)\} d\Omega(s|t_i).$$

Here $\Omega(.)$ is the CDF of $t_0 + k\eta + \tau^{ST}$ conditional on $t_i$, which is uniformly distributed over $[t_i - \eta + k\eta + \tau^{ST}, t_i + k\eta + \tau^{ST}]$. If $t_0 + k\eta + \tau^{ST} < t$, the regime already collapses when the small trader attacks, and the small trader gains 0. If $t_0 + k\eta + \tau^{ST} \geq t$, the regime collapses at $t_0 + k\eta + \tau^{ST}$, and the small trader gains $g(k\eta + \tau^{ST}) - c(t_0 + k\eta + \tau^{ST} - t)$. Therefore, we have the above expected payoff function.

(c) If $t < t_i - \eta + k\eta + \tau^{ST}$, $E(\Pi^{ST}|t_i) = \int_{t_i-\eta+k\eta+\tau^{ST}}^{t_i+k\eta+\tau^{ST}} \{g(k\eta + \tau^{ST}) - c(s - t)\} d\Omega(s|t_i)$.

This case is similar to the above one except that the small trader's attacking time $t$ is earlier than $t_i - \eta + k\eta + \tau^{ST}$. Thus the small trader always attacks before $t_0 + k\eta + \tau^{ST}$. In this case, the lower bound of the above integration is $t_i - \eta + k\eta + \tau^{ST}$, instead of $t$.

2. When $t_0 + (k - \lambda)\eta + \tau^{ST} \geq t_0 + \eta^{LT} + \tau^{LT}$, the regime collapses at $t_0 + (k - \lambda)\eta + \tau^{ST}$, and the small trader's profit is:

(a) If $t > t_i + (k - \lambda)\eta + \tau^{ST}$, $E(\Pi^{ST}|t_i) = 0$ since the regime already collapses when the small trader attacks.

(b) If $t_i - \eta + (k - \lambda)\eta + \tau^{ST} \leq t \leq t_i + (k - \lambda)\eta + \tau^{ST}$,

$$E(\Pi^{ST}|t_i) = \int_{t}^{t_i+(k-\lambda)\eta+\tau^{ST}} \{g[(k - \lambda)\eta + \tau^{ST}] - c(s - t)\} d\Psi(s|t_i).$$

Here $\Psi(.)$ is the CDF of $t_0 + (k - \lambda)\eta + \tau^{ST}$ conditional on $t_i$, which is uniformly distributed over $[t_i - \eta + (k - \lambda)\eta + \tau^{ST}, t_i + (k - \lambda)\eta + \tau^{ST}]$. If $t_0 + (k - \lambda)\eta + \tau^{ST} < t$, the regime already collapses when the small trader attacks, and the small trader gains 0. If $t_0 + (k - \lambda)\eta + \tau^{ST} \geq t$, the regime collapses at $t_0 + (k-\lambda)\eta + \tau^{ST}$, and the small trader gains $g[(k-\lambda)\eta+\tau^{ST}] - c(t_0+(k-\lambda)\eta+\tau^{ST}-t)$. Therefore, we have the above expected payoff function.

(c) If $t < t_i - \eta + (k-\lambda)\eta + \tau^{ST}$, $E(\Pi^{ST}|t_i) = \int_{t_i-\eta+(k-\lambda)\eta+\tau^{ST}}^{t_i+(k-\lambda)\eta+\tau^{ST}} \{g[(k-\lambda)\eta + \tau^{ST}] - c(s-t)\}\Psi(s|t_i)$.

This case is similar to the above one except that the small trader's attacking time $t$ is earlier than $t_i - \eta + (k-\lambda)\eta + \tau^{ST}$. Thus the lower bound of the above integration is $t_i - \eta + (k-\lambda)\eta + \tau^{ST}$, instead of $t$.

3. When $t_0+\tau^{LT} \le t_0+(k-\lambda)\eta+\tau^{ST} \le t_0+\eta^{LT}+\tau^{LT} \le t_0+k\eta+\tau^{ST}$, the regime may collapse at $t_0 + (k-\lambda)\eta + \tau^{ST}$ or at $t^{LT} + \tau^{LT}$, depending on when the large trader becomes aware of the overvaluation. The small trader's expected profit is

$$E(\Pi^{ST}|t_i) = \int_{t_i-\eta}^{t_i} f(t,s,x)d\Omega(x|t_i)$$

Here $\Omega(x|t_i)$ is the small trader's CDF of $t_0$ conditional on $t_i$, which is uniformly distributed over $[t_i-\eta, t_i]$. $f(t,s,x)$ gives the expected payoff of the small trader, given $t_0$ by attacking at $t$. Here $s$ represents $t^{LT} + \tau^{LT}$, the time when the large trader attacks, and $x$ represents $t_0$.

(a) when $t > x + \eta^{LT} + \tau^{LT}$, $f(t,s,x) = 0$ since the regime already collapses when the small trader attacks.

(b) When $x + (k-\lambda)\eta + \tau^{ST} < t \le x + \eta^{LT} + \tau^{LT}$,

$$f(t,s,x) = \int_t^{x+\eta^{LT}+\tau^{LT}} [g(s-x) - c(s-t)]d\Phi(s|x)$$

Here $\Phi(s|x)$ is the small trader's CDF of $t^{LT} + \tau^{LT}$ conditional on $t_0$, which is uniformly distributed over $[x + \tau^{LT}, x + \eta^{LT} + \tau^{LT}]$. In this case, if $t^{LT} + \tau^{LT} < t$, the small trader will gain zero since the regime already collapses when the small trader attacks. If $t^{LT} + \tau^{LT} \geq t$, the regime collapses at $t^{LT} + \tau^{LT}$, and the small trader gains $g(t^{LT} + \tau^{LT} - t_0) - c(t^{LT} + \tau^{LT} - t)$. Therefore, we get the above expected payoff function.

(c) When $x + \tau^{LT} \leq t \leq x + (k - \lambda)\eta + \tau^{ST}$,

$$\begin{aligned} f(t, s, x) = \int_{t}^{x+(k-\lambda)\eta+\tau^{ST}} & \{g[(k-\lambda)\eta + \tau^{ST}] - c[x + (k-\lambda)\eta + \tau^{ST} - t]\} d\Phi(s|x) \\ & + \int_{x+(k-\lambda)\eta+\tau^{ST}}^{x+\eta^{LT}+\tau^{LT}} [g(s-x) - c(s-t)] d\Phi(s|x) \end{aligned}$$

In this case, if $t^{LT} + \tau^{LT} < t_0 + (k - \lambda)\eta + \tau^{ST}$, the regime collapses at $t_0 + (k - \lambda)\eta + \tau^{ST}$. The small trader will gain $g[(k - \lambda)\eta + \tau^{ST}] - c(t_0 + (k - \lambda)\eta + \tau^{ST} - t)$. If $t^{LT} + \tau^{LT} \geq t_0 + (k - \lambda)\eta + \tau^{ST}$, the regime collapses at $t^{LT} + \tau^{LT}$, and the small trader gains $g(t^{LT} + \tau^{LT} - t_0) - c(t^{LT} + \tau^{LT} - t)$. Therefore, we get the above expected payoff function.

(d) When $t < x + \tau^{LT}$,

$$\begin{aligned} f(t, s, x) = \int_{x+\tau^{LT}}^{x+(k-\lambda)\eta+\tau^{ST}} & \{g[(k-\lambda)\eta + \tau^{ST}] - c[x + (k-\lambda)\eta + \tau^{ST} - t]\} d\Phi(s|x) \\ & + \int_{x+(k-\lambda)\eta+\tau^{ST}}^{x+\eta^{LT}+\tau^{LT}} [g(s-x) - c(s-t)] d\Phi(s|x) \end{aligned}$$

This case is similar to the above case. The only difference is that now the small trader's attacking time $t$ is earlier than $t_0 + \tau^{LT}$. Therefore, the first item of the above expected payoff function has the lower bound of $x + \tau^{LT}$.

4. When $t_0 + \tau^{LT} \leq t_0 + (k - \lambda)\eta + \tau^{ST} < t_0 + k\eta + \tau^{ST} \leq t_0 + \eta^{LT} + \tau^{LT}$, the regime may collapse at $t_0 + (k - \lambda)\eta + \tau^{ST}$, $t^{LT} + \tau^{LT}$ or $t_0 + k\eta + \tau^{ST}$, depending

on how $t^{LT} + \tau^{LT}$ is realized. The small trader's expected profit is given by:

$$E(\Pi^{ST}|t_i) = \int_{t_i-\eta}^{t_i} f(t,s,x)d\Omega(x|t_i)$$

Here $\Omega(x|t_i)$ and $f(t,s,x)$ are the same as the ones in the above case.

(a) If $t > x + k\eta + \tau^{ST}$, $f(t,s,x) = 0$ since the regime already collapses when the small trader attacks.

(b) If $x + (k-\lambda)\eta + \tau^{ST} < t < x + k\eta + \tau^{ST}$, $f(t,s,x) = \int_t^{x+k\eta+\tau^{ST}} [g(s-x) - c(s-t)]d\Phi(s|x) + \int_{x+k\eta+\tau^{ST}}^{x+\eta^{LT}+\tau^{LT}} [g(k\eta+\tau^{ST}) - c(x+k\eta+\tau^{ST}-t)]d\Phi(s|x)$.

Here $\Phi(s|x)$ is the small trader's CDF of $t^{LT} + \tau^{LT}$ conditional on $t_0$, which is uniformly distributed over $[t_0 + \tau^{LT}, t_0 + \eta^{LT} + \tau^{LT}]$. In this case, if $t^{LT} + \tau^{LT} < t$, the small trader will gain zero since the regime already collapses when the small trader attacks. If $t \le t^{LT} + \tau^{LT} \le t_0 + k\eta + \tau^{ST}$, the regime collapses at $t^{LT} + \tau^{LT}$, and the small trader gains $g(t^{LT} + \tau^{LT} - t_0) - c(t^{LT} + \tau^{LT} - t)$. If $t^{LT} + \tau^{LT} \ge t_0 + k\eta + \tau^{ST}$, the small trader gains $g(k\eta + \tau^{ST}) - c(t_0 + k\eta + \tau^{ST} - t)$. Therefore, we get the above expected payoff function.

(c) If $x + \tau^{LT} < t < x + (k-\lambda)\eta + \tau^{ST}$, $f(t,s,x) = \int_t^{x+(k-\lambda)\eta+\tau^{ST}} g((k-\lambda)\eta + \tau^{ST}) - c(x + (k-\lambda)\eta + \tau^{ST} - t)d\Phi(s|x) + \int_{x+(k-\lambda)\eta+\tau^{ST}}^{x+k\eta+\tau^{ST}} g(s-x) - c(s-t)d\Phi(s|x) + \int_{x+k\eta+\tau^{ST}}^{x+\eta^{LT}+\tau^{LT}} [g(k\eta+\tau^{ST}) - c(x+k\eta+\tau^{ST}-t)]d\Phi(s|x)$

This case is similar to the above one except that now the small trader's attacking time $t$ is earlier than $t_0 + (k-\lambda)\eta + \tau^{ST}$. Thus if $t^{LT} + \tau^{LT} < t_0 + (k-\lambda)\eta + \tau^{ST}$, the small trader gains $g[(k-\lambda)\eta + \tau^{ST}] - c(t_0 + (k-\lambda)\eta + \tau^{ST} - t)$. The rest will be similar to the above case.

(d) If $t < x+\tau^{LT}$, $f(t,s,x) = \int_{x+\tau^{LT}}^{x+(k-\lambda)\eta+\tau^{ST}} g((k-\lambda)\eta+\tau^{ST})-c(x+(k-\lambda)\eta+\tau^{ST}-t)d\Phi(s|x)+\int_{x+(k-\lambda)\eta+\tau^{ST}}^{x+k\eta+\tau^{ST}} g(s-x)-c(s-t)d\Phi(s|x)+\int_{x+k\eta+\tau^{ST}}^{x+\eta^{LT}+\tau^{LT}} [g(k\eta+\tau^{ST}) - c(x+k\eta+\tau^{ST}-t)]d\Phi(s|x)$.

This case is similar to the above one except that now the small trader's attacking time $t$ is earlier than $t_0+\tau^{LT}$. Thus the lower bound of the first item in the above function is $x+\tau^{LT}$, instead of $t$.

5. When $t_0+(k-\lambda)\eta+\tau^{ST} \leq t_0+\tau^{LT} < t_0+\eta^{LT}+\tau^{LT} \leq t_0+k\eta+\tau^{ST}$, the regime collapses exactly at $t^{LT}+\eta^{LT}$ when the large trader attacks. Thus, the small trader's expected profit is:

$$E(\Pi^{ST}|t_i) = \int_{t_i-\eta}^{t_i} f(t,s,x)d\Omega(x|t_i)$$

Here $\Omega(x|t_i)$ and $f(t,s,x)$ are the same as the ones in the above case.

(a) If $t > x+\eta^{LT}+\tau^{LT}$, $f(t,s,x) = 0$, since the regime already collapses when the small trader attacks.

(b) If $x+\tau^{LT} \leq t \leq x+\eta^{LT}+\tau^{LT}$, $f(t,s,x) = \int_{t}^{x+\eta^{LT}+\tau^{LT}} [g(s-x)-c(s-t)]d\Phi(s|x)$.

Here $\Phi(s|x)$ is the CDF of $t^{LT}+\tau^{LT}$ $(s)$ conditional on $t_0$ $(x)$. In this case, the regime collapses at $t^{LT}+\tau^{LT}$, and the small trader gains $g[t^{LT}+\tau^{LT}-t_0]-c(t^{LT}+\tau^{LT}-t)$.

(c) If $t < x+\tau^{LT}$, $f(t,s,x) = \int_{x+\tau^{LT}}^{x+\eta^{LT}+\tau^{LT}} [g(s-x)-c(s-t)]d\Phi(s|x)$.

This case is similar to the above one except that now the small trader's attacking time $t$ is earlier than $t_0+\tau^{LT}$. Thus the lower bound of the above function is $x+\tau^{LT}$, instead of $t$.

6. When $t_0 + (k-\lambda)\eta + \tau^{ST} \leq t_0 + \tau^{LT} \leq t_0 + k\eta + \tau^{ST} \leq t_0 + \eta^{LT} + \tau^{LT}$, the regime collapses at $t^{LT} + \tau^{LT}$ or $t_0 + k\eta + \tau^{ST}$, depending on how $t^{LT} + \eta^{LT}$ is realized. The small trader's expected profit is:

$$E(\Pi^{ST}|t_i) = \int_{t_i-\eta}^{t_i} f(t,s,x) d\Omega(x|t_i)$$

Here $\Omega(x|t_i)$ and $f(t,s,x)$ are the same as the ones in the above cases.

(a) If $t > x + k\eta + \tau^{ST}$, $f(t,s,x) = 0$ since the regime already collapses when the small trader attacks.

(b) If $x + \tau^{LT} < t < x + k\eta + \tau^{ST}$, $f(t,s,x) = \int_t^{x+k\eta+\tau^{ST}} [g(s-x) - c(s-t)] d\Phi(s|x) + \int_{x+k\eta+\tau^{ST}}^{x+\eta^{LT}+\tau^{LT}} [g(k\eta + \tau^{ST}) - c(x + k\eta + \tau^{ST} - t)] d\Phi(s|x)$.

$\Phi(s|x)$ is the small trader's CDF of $t^{LT} + \tau^{LT}$ conditional on $t_0$, which is uniformly distributed over $[t_0 + \tau^{LT}, t_0 + \eta^{LT} + \tau^{LT}]$. In this case, if $t^{LT} + \tau^{LT} < t$, the small trader will gain zero since the regime already collapses when the small trader attacks. If $t_0 + k\eta + \tau^{ST} \geq t^{LT} + \tau^{LT} \geq t$, the regime collapses at $t^{LT} + \tau^{LT}$, and the small trader gains $g(t^{LT} + \tau^{LT} - t_0) - c(t^{LT} + \tau^{LT} - t)$. If $t^{LT} + \tau^{LT} \geq t_0 + k\eta + \tau^{ST}$, the small trader gains $g(k\eta + \tau^{ST}) - c(t_0 + k\eta + \tau^{ST} - t)$. Therefore, we get the above expected profit function.

(c) If $t < x + \tau^{LT}$, $f(t,s,x) = \int_{x+\tau^{LT}}^{x+k\eta+\tau^{ST}} [g(s-x) - c(s-t)] d\Phi(s|x) + \int_{x+k\eta+\tau^{ST}}^{x+\eta^{LT}+\tau^{LT}} [g(k\eta + \tau^{ST}) - c(x + k\eta + \tau^{ST} - t)] d\Phi(s|x)$.

This case is similar to the above one except that now the small trader's attacking time $t$ is earlier than $t_0 + \tau^{LT}$. Thus the lower bound of the first item in the above function is $x + \tau^{LT}$, instead of $t$.

The small trader will choose his attacking time to maximize his expected profits. Since this is a symmetric strategy equilibrium, $\tau^* = \tau^{ST}$ in equilibrium. Thus we find the best response of small traders as a function of $\tau^{LT}$.

It is difficult to find analytical solutions to the best responses of small traders in each case due to the properties of the expected profit function of small traders. Numerical analysis will be given in Section 5.2.

We have two equations: $\tau^{LT}$ as a function of $\tau^{ST}$, and $\tau^{ST}$ as a function of $\tau^{LT}$. Solving these two equations, we find the equilibrium strategies for both the large and small traders. In the following numerical examples, we demonstrate that with reasonable parameter values, there exists a unique Bayesian Nash equilibrium in this game.

## 5.2 Numerical Results

In this section, we will give some numerical examples to study the equilibrium in this model. We assume that $k = 0.9$, $\lambda = 0.5$, $\eta = 50$, $\eta^{LT} = 25$, $g = 0.01$, and $c = 0.015$ as the benchmark. Then we will conduct comparative statics practice on $\lambda$ and $\eta^{LT}$ to examine how the size of the large trader' wealth and the precision of the large trader's information will affect equilibrium outcomes. Here we do not intend to calibrate the economy. Instead, we only attempt to demonstrate qualitatively how $\lambda$ and $\eta^{LT}$ affect the economy. Here we choose $c > g$ since it is a key condition to induce traders to "ride the overvaluation" and to delay their attack after becoming aware of the overvaluation in our model due to the setup of the model.

### 5.2.1 An Example

We study the equilibrium using a numerical example with the parameter values specified at the beginning of this section.

First, we study how the equilibrium strategy of the large trader, $\tau^{LT}$, changes with small traders' strategies $\tau^{ST}$.

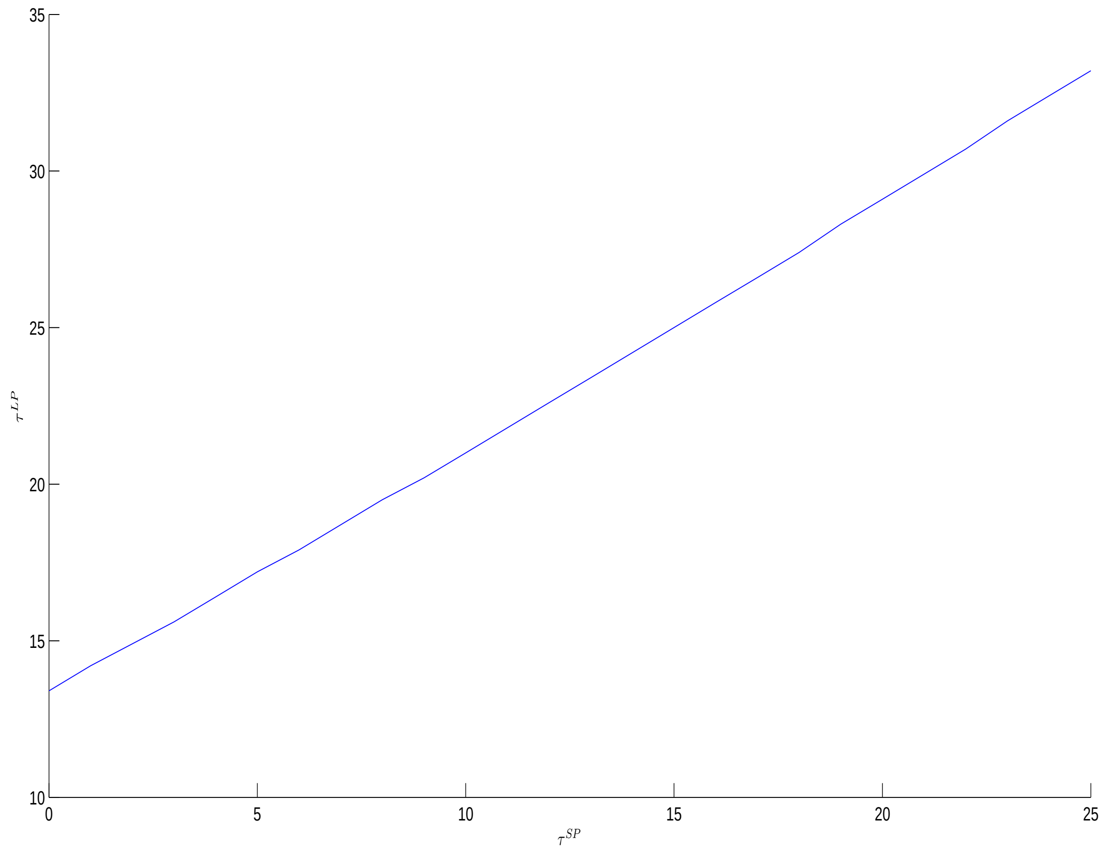


Figure 6: How the Large Player's Best Response $\tau^{LT}$ Changes in $\tau^{ST}$

From Figure 6 we can tell that the large trader's strategy, $\tau^{LT}$ is strictly increasing in small trader's strategy $\tau^{ST}$. In addition, there is a seemingly linear relationship between $\tau^{LT}$ and $\tau^{ST}$. Further analytical analysis reveals that the slope of the curve is close to 1 but not constant, slightly varying in $\tau^{ST}$. More specifically, with the parameter values in our numerical example, the optimal $\tau^{LT}$ that maximizes the large trader's expected payoff is given by (see Appendix B for a detailed derivation):

$$\tau^{LT} = a^* - \eta^{LT} + (k - \lambda)\eta + \tau^{ST},$$

where $a^*$ is given by

$$a^* = \frac{-w + \sqrt{w^2 + \frac{4c}{g}\lambda\eta\eta^{LT}}}{2},$$

where $w = k\eta + \frac{c}{g}\lambda\eta + \tau^{ST} - 2\lambda\eta$.

Second, we examine how the equilibrium strategy of small traders, $\tau^{ST}$, changes with the large trader's strategy $\tau^{LT}$.

From Figure 7 we find that:

1. When $\tau^{LT}$ is extremely small, case 2 is realized. In this case, the regime will collapse at $t_0 + (k - \lambda)\eta + \tau^{ST}$, and the small trader's equilibrium strategy $\tau^{ST} = \frac{c-g}{g}(k - \lambda)\eta = 10$, which is irrelevant to $\tau^{LT}$.

2. When $\tau^{LT}$ gradually increases from 0, case 3 is realized. In this case, the regime may collapse at $t_0 + (k - \lambda)\eta + \tau^{ST}$ or at $t^{LT} + \tau^{LT}$, depending on when the large trader becomes aware of the overvaluation. The small trader's equilibrium strategy $\tau^{ST}$ increases in the large trader's strategy $\tau^{LT}$. This is because the collapsing time of the regime now depends on $t^{LT} + \tau^{LT}$, and a larger $\tau^{LT}$ will delay the collapse of the regime and subsequently induce small traders to delay their attack.

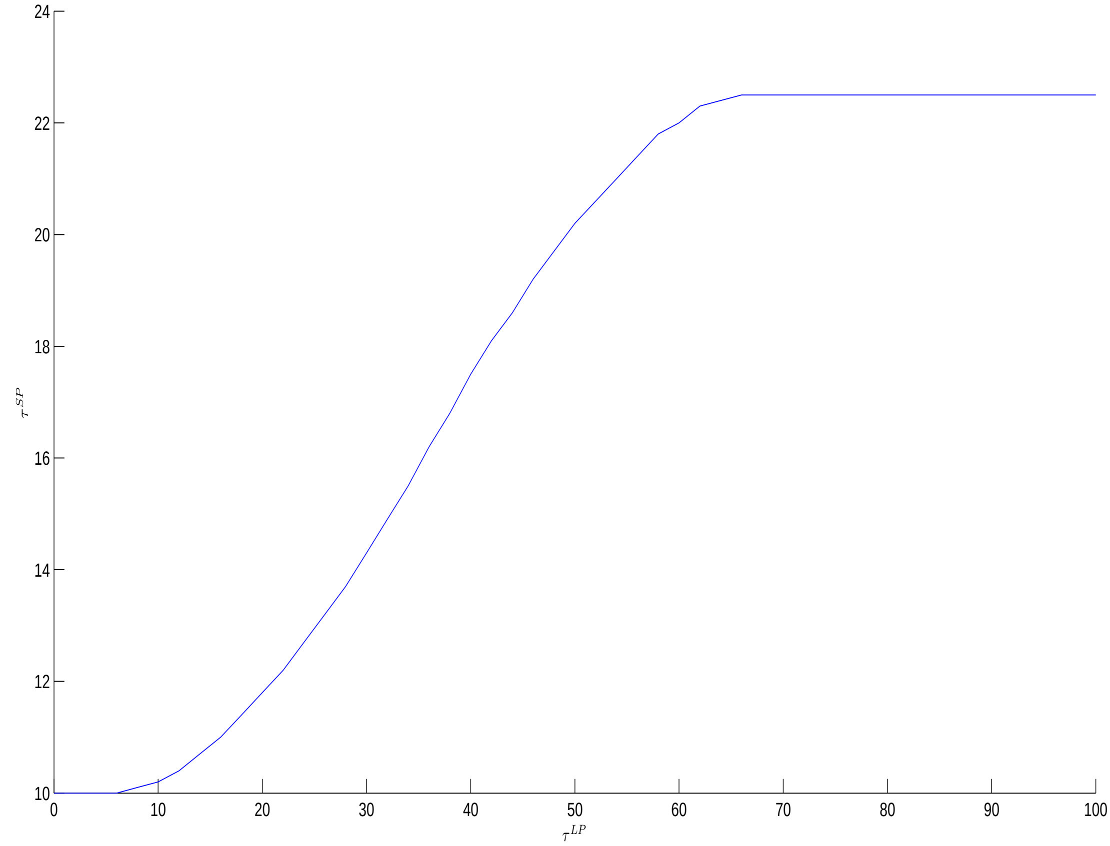


Figure 7: How Small Players' Best Response $\tau^{ST}$ Changes in $\tau^{LT}$

3. When $\tau^{LT}$ increases further, case 6 is realized. In this case, the regime collapses at $t^{LT} + \tau^{LT}$ or $t_0 + k\eta + \tau^{ST}$, depending on how $t^{LT} + \eta^{LT}$ is realized. Similarly, $\tau^{ST}$ will increase in $\tau^{LT}$ here.

4. When $\tau^{LT}$ is extremely large, case 1 is realized. In this case, the regime will collapse at $t_0 + k\eta + \tau^{ST}$, and the small trader's equilibrium strategy $\tau^{ST} = \frac{c-g}{g} k\eta = 22.5$, which is irrelevant to $\tau^{LT}$.

The equilibrium in this example is given by the intersection of the best responses of the large trader and small traders in Figure 8. From Figure 8 we can see that there

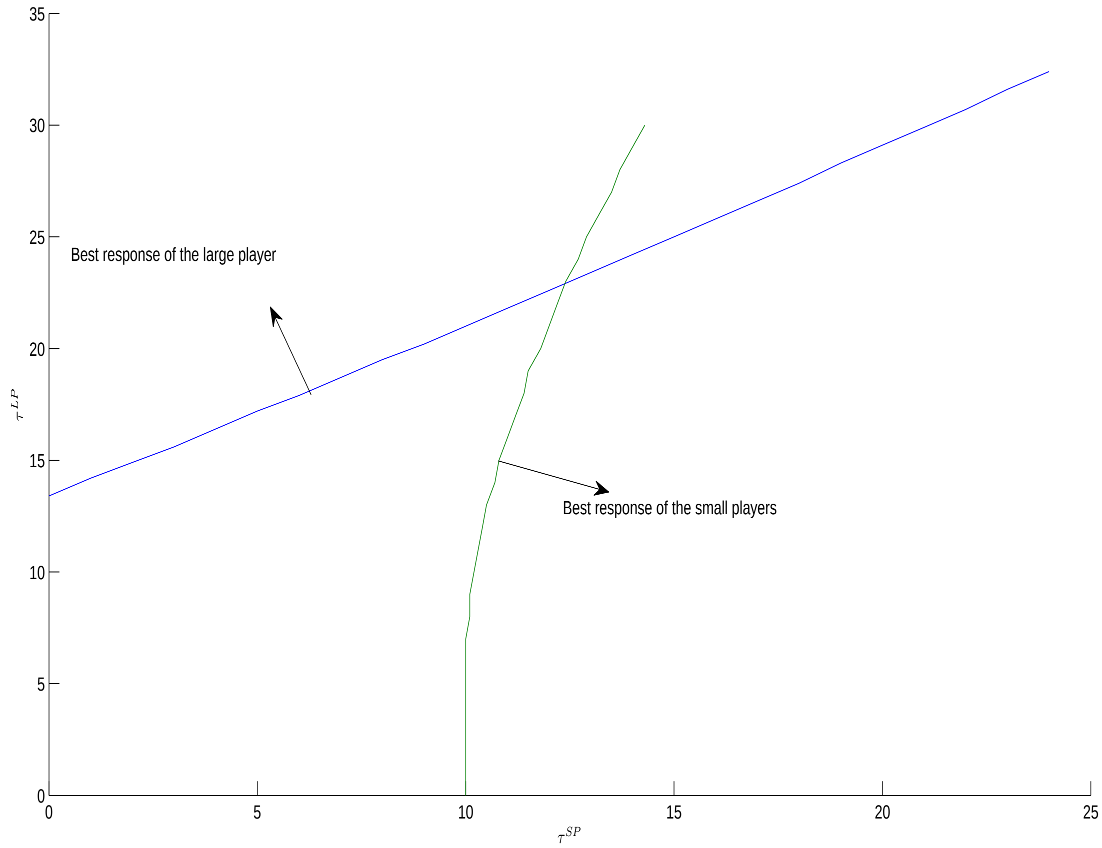


Figure 8: The Equilibrium in the Example

is a unique equilibrium in our example. In this equilibrium, the large trader will attack some periods between 20 and 25 after he becomes aware of the overvaluation. Small traders will attack between period 10 and 15 after they become aware of the overvaluation. Depending on exactly when the large trader becomes aware of the overvaluation, the regime collapses at $t_0 + (k - \lambda)\eta + \tau^{ST}$ or $t^{LT} + \tau^{LT}$.

Further calculation tells us that in this example, in equilibrium $\tau^{ST} = 12.41$ and $\tau^{LT} = 22.91$. That is, in equilibrium, each small trader will launch an attack 12.41

periods after he becomes aware of the overvaluation. While the large trader will launch the attack 22.91 periods after he becomes aware of the overvaluation. The regime will collapse at $t^{LT} + \tau^{LT}$ if $t^{LT} \in [t_0 + 9.5, t_0 + 25]$. If $t^{LT} \in [t_0, t_0 + 9.5]$, the regime collapses at $t_0 + (k - \lambda)\eta + \tau^{ST}$, or $t_0 + 32.41$. We know that ex ante $t^{LT}$ is uniformly distributed over $[t_0, t_0 + 25]$. Therefore, with probability of 38%, the regime will collapse at $t_0 + 32.41$. With probability of 62%, the regime will collapse at any moment between $t_0 + 32.41$ and $t_0 + 47.91$.

Recall that in a market without a large trader, the collapsing time of the regime is perfectly predictable given $t_0$ is known. However, in the presence of the large trader, we find that the collapsing time of the regime becomes quite uncertain, and crucially depends on when the large trader becomes aware of the overvaluation. Now given $t_0$, we only know the distribution of the time the regime collapses. Therefore, it becomes much more difficult for traders to time the collapse of the regime. In this sense, our model demonstrates that the feature of predictability of the time of collapse given $t_0$ in a model only with small traders is quite fragile. It is rather sensitive to the information structure in the model. Thus our model reveals that in a real world with a much more complicated information structure than in our model, it will be extremely difficult for traders to time and profit from a bubble.

### 5.2.2 How the Equilibrium Changes in $\lambda$

Now we examine how the size of the wealth of the large trader, $\lambda$, will influence the dynamics of a currency attack. We find that with larger size of the large trader's wealth, both the best response of the large trader $\tau^{LT}$ and that of small traders $\tau^{ST}$ will be smaller. A larger $\lambda$ will accelerate the collapse of the regime greatly.

Figure 9 shows that the large trader's strategy, $\tau^{LT}$, is strictly decreasing in his size of wealth, $\lambda$. The intuition for this result is that the large trader will choose some moment after $t_0 + (k - \lambda)\eta + \tau^{ST}$ to attack. With larger $\lambda$, $t_0 + (k - \lambda)\eta + \tau^{ST}$ is lower, and the attacking moment of the large trader tends to be lower too.

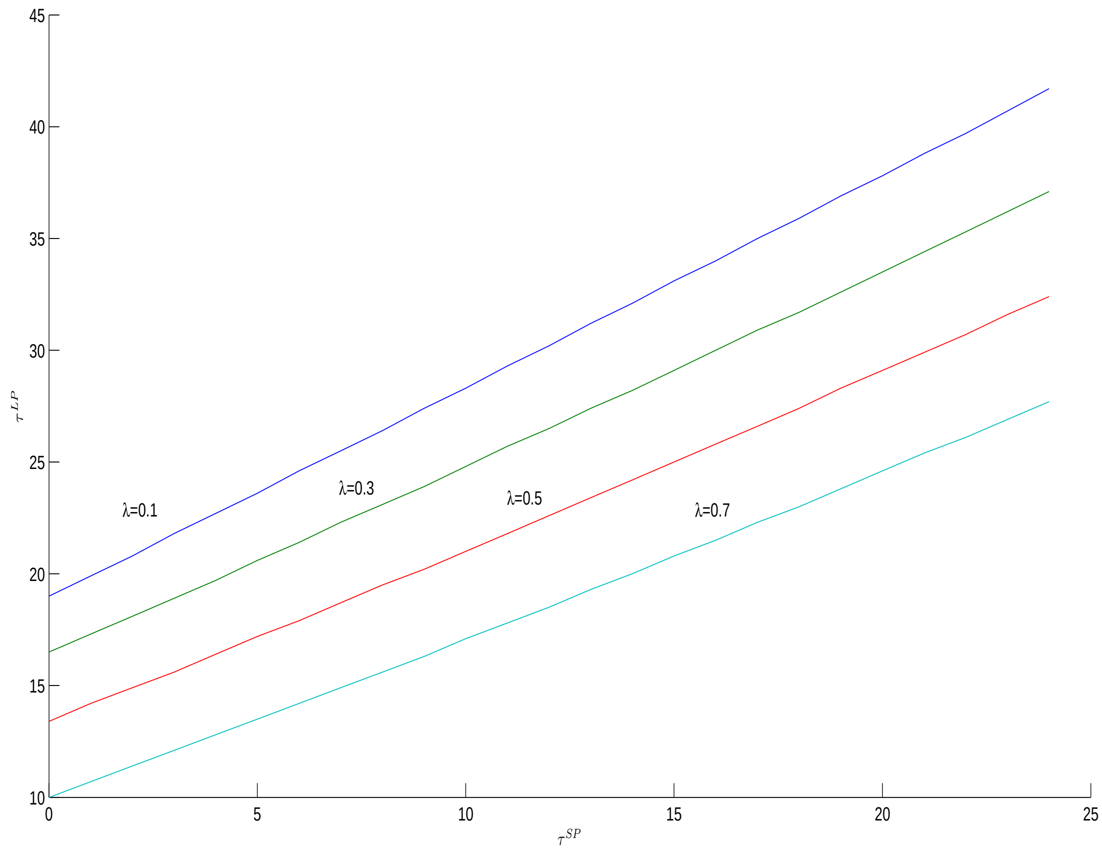


Figure 9: How the Large Player's Best Response $\tau^{LT}$ Changes in $\lambda$

Figure 10 shows how small traders' equilibrium strategy $\tau^{ST}$ changes with $\lambda$. We find that the larger $\lambda$, the more often that $\tau^{ST}$ falls into the range in which it is increasing in $\tau^{LT}$. That is, $\tau^{ST}$ is more sensitive to the changes in $\tau^{LT}$. In addition, we find that with larger $\lambda$, $\tau^{ST}$ will be lower. That is, small traders will be more aggressive and attack sooner.

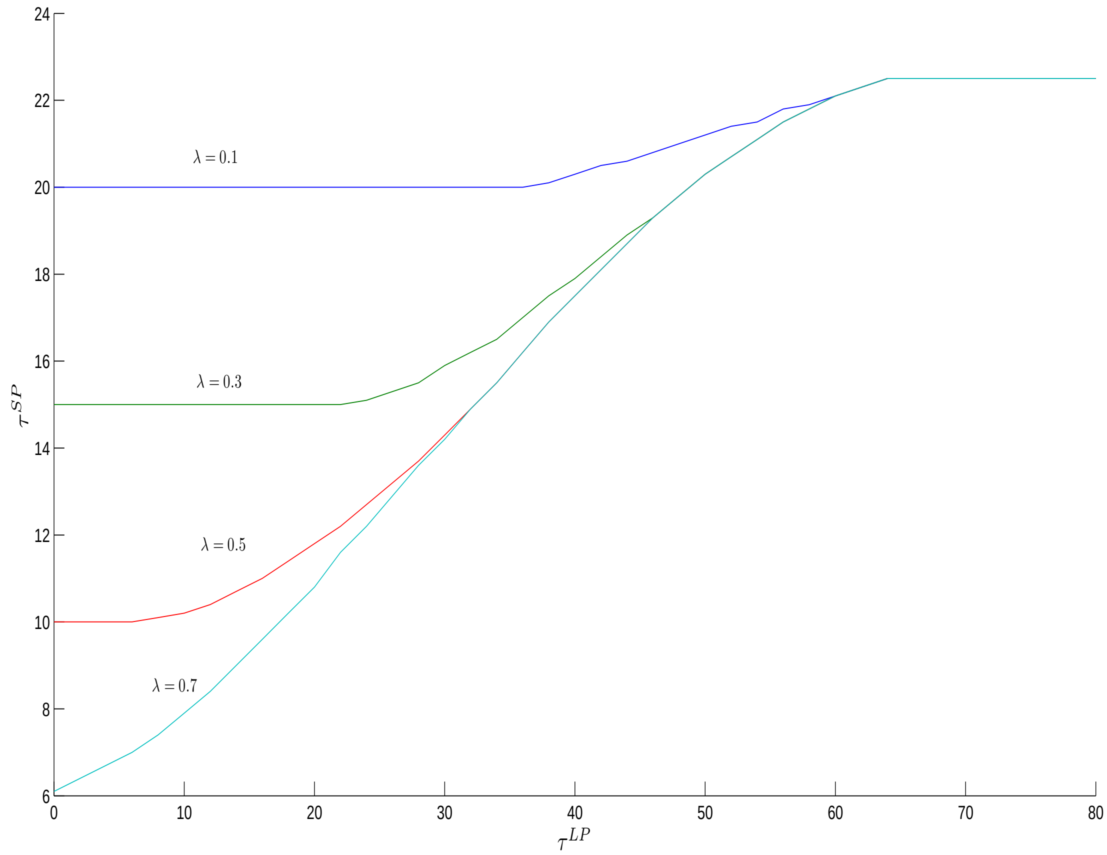


Figure 10: How Small Players' Best Response $\tau^{ST}$ Changes in $\lambda$

The intuition for Figure 10 is as follows. Recall that with different parameter values, there are six different combinations and each will give a different expected profit function of the small trader. Consequently the small trader will have different best response under each combination. We find that:

1. When $\lambda$ is extremely large ($\lambda = 0.7$), with $\tau^{LT}$ increasing from 0, $\tau^{LT} < (k - \lambda)\eta + \tau^{ST} < \eta^{LT} + \tau^{LT} < k\eta + \tau^{ST}$, $(k - \lambda)\eta + \tau^{ST} < \tau^{LT} < \eta^{LT} + \tau^{LT} < k\eta + \tau^{ST}$,

$(k-\lambda)\eta+\tau^{ST} < \tau^{LT} < k\eta+\tau^{ST} < \eta^{LT}+\tau^{LT}$, and $\tau^{LT} > k\eta+\tau^{ST}$ are realized sequentially.

2. When $\lambda = 0.5$ or $0.3$, with $\tau^{LT}$ increasing from 0, $\tau^{LT}+\eta^{LT} < (k-\lambda)\eta+\tau^{ST}$, $\tau^{LT} < (k-\lambda)\eta+\tau^{ST} < \eta^{LT}+\tau^{LT} < k\eta+\tau^{ST}$, $(k-\lambda)\eta+\tau^{ST} < \tau^{LT} < k\eta+\tau^{ST} < \eta^{LT}+\tau^{LT}$, and $\tau^{LT} > k\eta+\tau^{ST}$ are realized sequentially.

3. When $\lambda$ is extremely small ($\lambda = 0.1$), with $\tau^{LT}$ increasing from 0, $\tau^{LT}+\eta^{LT} < (k-\lambda)\eta+\tau^{ST}$, $\tau^{LT} < (k-\lambda)\eta+\tau^{ST} < \eta^{LT}+\tau^{LT} < k\eta+\tau^{ST}$, $\tau^{LT} < (k-\lambda)\eta+\tau^{ST} < k\eta+\tau^{ST} < \eta^{LT}+\tau^{LT}$, $(k-\lambda)\eta+\tau^{ST} < \tau^{LT} < k\eta+\tau^{ST} < \eta^{LT}+\tau^{LT}$, and $\tau^{LT} > k\eta+\tau^{ST}$ are realized sequentially.

We find that when $\lambda$ equals 0.1, 0.3 and 0.5, the case of $\tau^{LT}+\eta^{LT} < (k-\lambda)\eta+\tau^{ST}$ is realized when $\tau^{LT}$ starts from 0. We know that in this case, the regime collapses at $(k-\lambda)\eta+\tau^{ST}$, which is irrelevant to the large trader's strategy. That is why we observe the flat line in all the three cases. In addition, from last section, we know that $\tau^{ST*} = \frac{c-g}{g}(k-\lambda)\eta$. In our numerical examples, $\tau^{ST*}$ will be 20 (when $\lambda = 0.1$), 15(when $\lambda = 0.3$), and 10(when $\lambda = 0.5$). This analytical result is consistent with our numerical one.

Moreover, we find that in all the above four examples, the case of $\tau^{LT} > k\eta+\tau^{ST}$ will be realized when $\tau^{LT}$ is large enough. In this case, the regime collapses at $k\eta+\tau^{ST}$, which is irrelevant to both $\tau^{LT}$ and $\lambda$. That is why we observe the overlapping flat lines in all the four example. Analytically we know that $\tau^{ST*} = \frac{c-g}{g}k\eta$, which will be 22.5 in our numerical examples. This analytical result is consistent with our numerical one.

Figure 11 shows how the equilibrium shifts when $\lambda$ changes. We find that with higher $\lambda$, both $\tau^{ST}$ and $\tau^{LT}$ are smaller in equilibrium. That is, with larger size of the wealth of the large trader, both the large trader and small traders are more aggressive and attack sooner. The intuition is straightforward. With larger $\lambda$, both best responses of the large trader and small traders will be more aggressive, leading to more aggressive equilibrium outcomes. In addition, we find that a larger $\lambda$ greatly accelerates the collapse of the regime. In our four examples, when $\lambda$ equals 0.7, 0.5 and 0.3, the regime may collapses at $t_0 + (k - \lambda)\eta + \tau^{ST}$ or later, depending when $t^{LT}$ is realized. The earliest collapsing time of the regime is reduced when $\lambda$ is larger not only because $\tau^{ST}$ is smaller, but also because $(k - \lambda)\eta$ is smaller.

### 5.2.3 How the Equilibrium Changes in $\eta^{LT}$

Here we examine how the precision of the large trader's information will affect the dynamics of a currency attack. We find that with more precise information of the large trader leads to smaller $\tau^{LT}$ and larger $\tau^{ST}$.

Figure 12 shows that the large trader's strategy, $\tau^{LT}$ is strictly decreasing in $\eta^{LT}$. Thus, the more precise the large trader's information is, the longer the large trader will delay his attack after he becomes aware of the overvaluation. The intuition here is that the more precise the large trader's information is, the better he can time the collapse of the regime, and the longer he can delay his attack.

Figure 13 shows how small traders' equilibrium strategy $\tau^{ST}$ changes with $\eta^{LT}$, the precision of the large trader's information.

From Figure 13, we can see that the larger $\eta^{LT}$ is, the more often that $\tau^{ST}$ falls into the range that it is increasing in $\tau^{LT}$. That is, $\tau^{ST}$ is more sensitive to the changes in $\tau^{LT}$. Moveover, we find that with lower $\eta^{LT}$, $\tau^{ST}$ will be lower. That is,

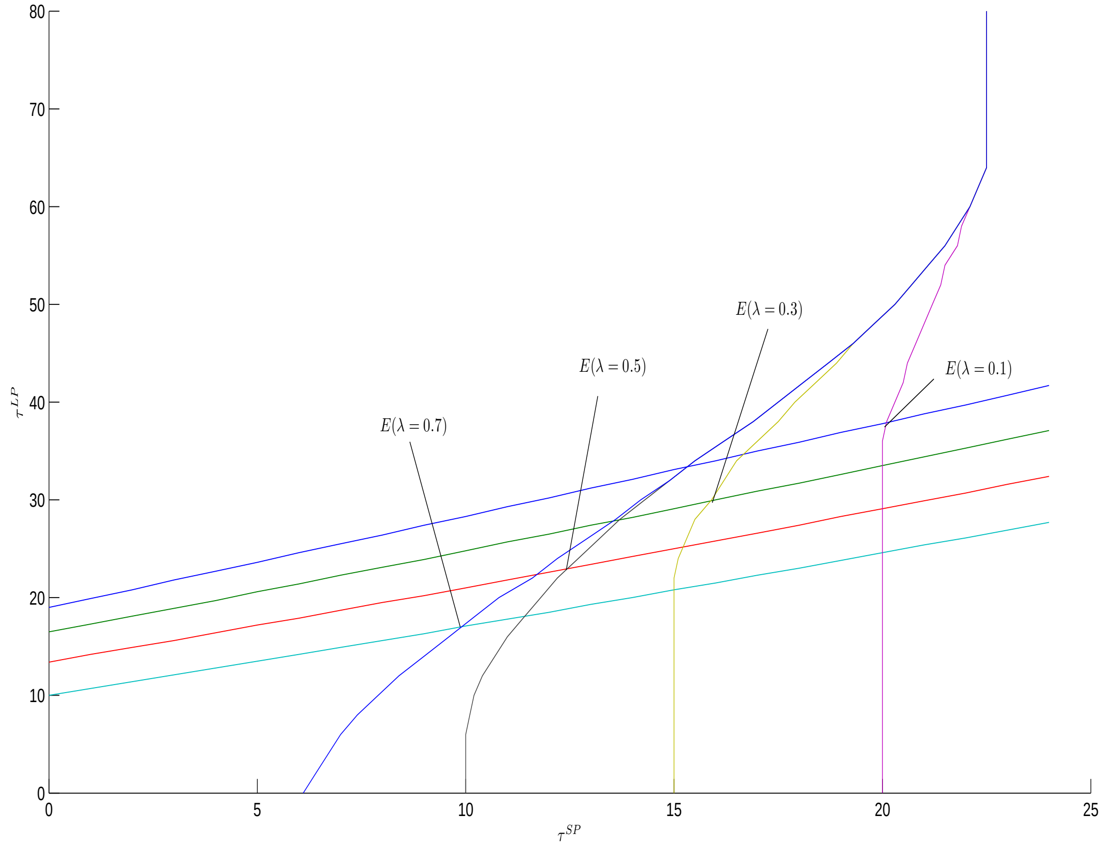


Figure 11: How the Equilibrium Changes in $\lambda$

small traders will be more aggressive and attack sooner. The intuition behind Figure 13 is as follows:

1. When $\tau^{LT}$ is extremely small, $\tau^{LT} + \eta^{LT} < (k-\lambda)\eta + \tau^{ST}$, and the regime collapses at $t_0 + (k-\lambda)\eta + \tau^{ST}$. The smaller $\eta^{LT}$ is, the more this case is possible. This explains that the curve of $\eta^{LT} = 5$ has the longest flat line part when $\tau^{LT}$ increases from zero. Moreover, when the regime collapses at $t_0 + (k-\lambda)\eta + \tau^{ST}$, $\tau^{ST*} = \frac{c-g}{g}(k-\lambda)\eta = 10$, which is irrelevant to $\eta^{LT}$. This

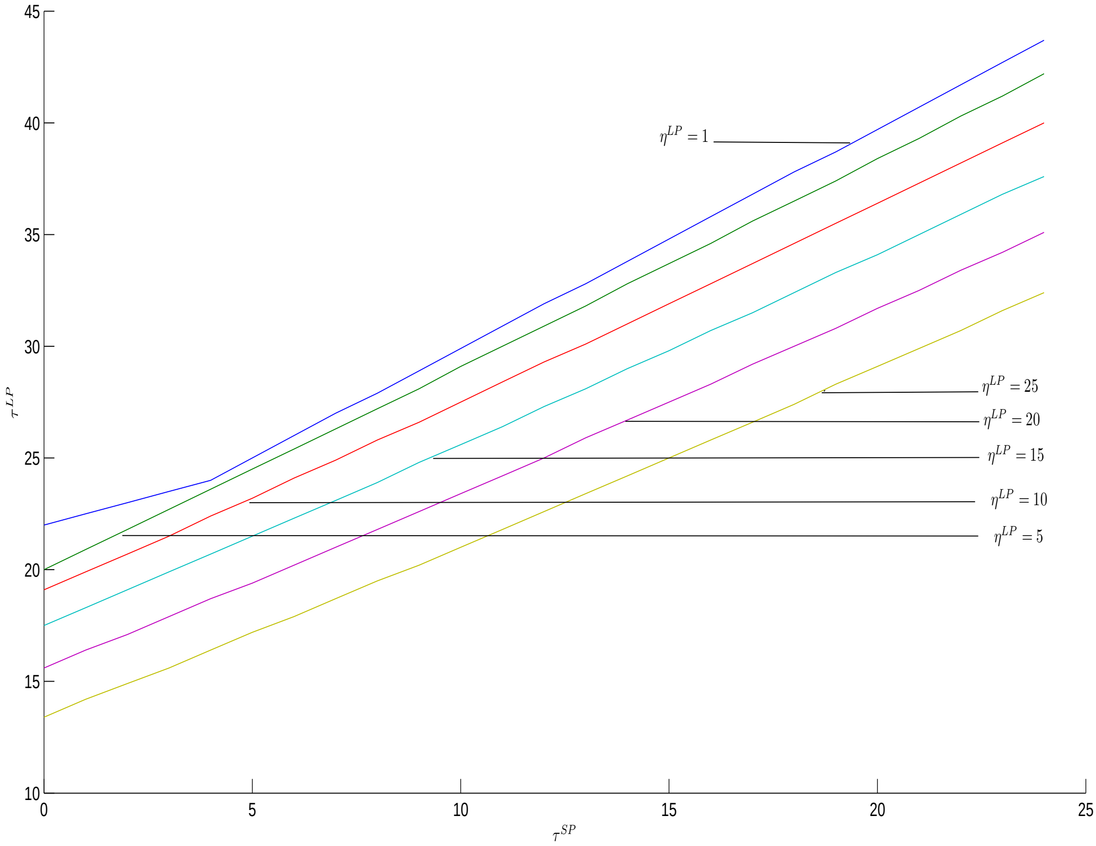


Figure 12: How the Large Player's Best Response $\tau^{LT}$ Changes in $\eta^{LT}$

explains the overlapping flat part of all curves when $\tau^{LT}$ increases from zero.

2. With the increase in $\tau^{LT}$, $\tau^{LT} < (k-\lambda)\eta + \tau^{ST} < \tau^{LT} + \eta^{LT} < k\eta + \tau^{ST}$. In this case, the regime may collapses at $t_0 + (k-\lambda)\eta + \tau^{ST}$ or $t^{LT} + \tau^{LT}$, where $t^{LT} \in [t_0, t_0 + \eta^{LT}]$. With lower $\eta^{LT}$, the possible collapsing time $t^{LT} + \tau^{LT}$ will be smaller too. This explains why $\tau^{ST}$ is lower with lower $\eta^{LT}$ in this case.

3. When $\tau^{LT}$ keeps increasing, $(k-\lambda)\eta + \tau^{ST} < \tau^{LT} < \tau^{LT} + \eta^{LT} < k\eta + \tau^{ST}$. In this case, the regime collapses at $t^{LT} + \tau^{LT}$ for sure. $\tau^{ST}$ is lower with lower

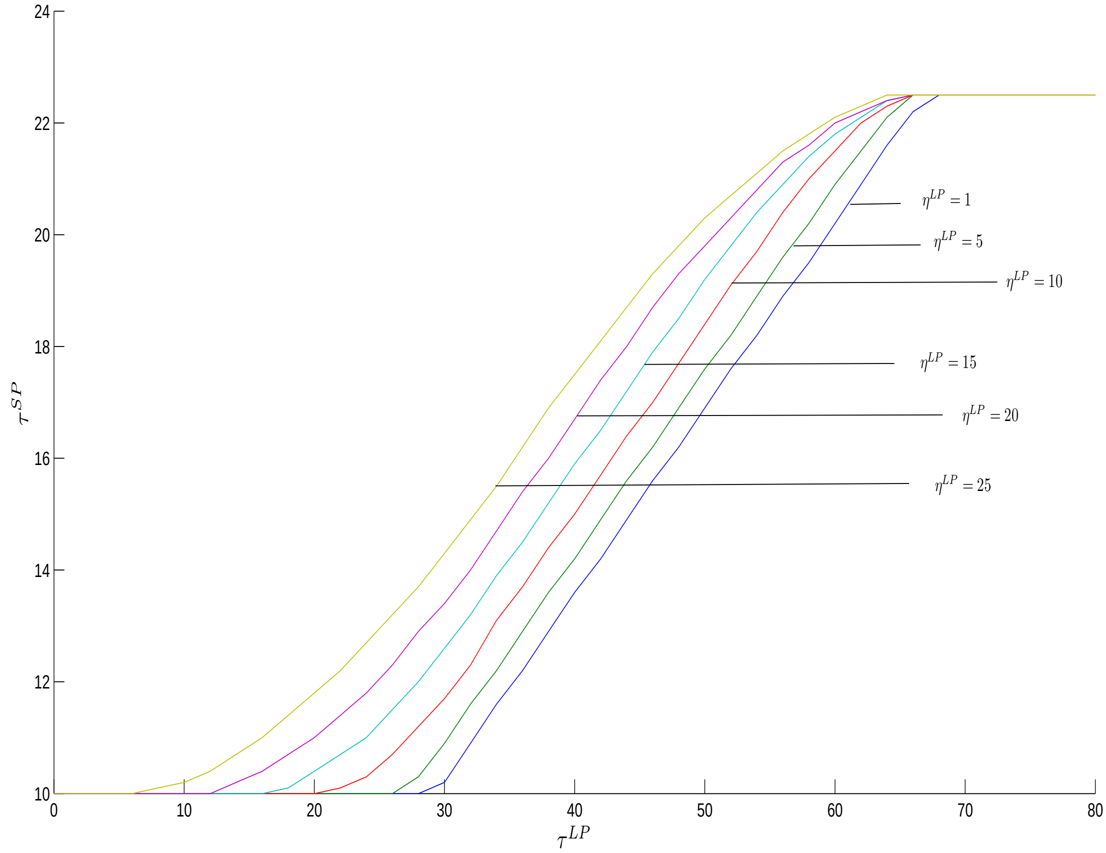


Figure 13: How Small Players' Best Response $\tau^{ST}$ Changes in $\eta^{LT}$

$\eta^{LT}$ for the similar reason we mentioned above.

4. When $\tau^{LT}$ keeps increasing, $(k-\lambda)\eta + \tau^{ST} < \tau^{LT} < k\eta + \tau^{ST} < \tau^{LT} + \eta^{LT}$. In this case, the regime may collapse at $t^{LT} + \tau^{LT}$ or $t_0 + k\eta + \tau^{ST}$. $\tau^{ST}$ is lower with lower $\eta^{LT}$ for the similar reason we mentioned above.

5. When $\tau^{LT}$ is extremely large, $\tau^{LT} > k\eta + \tau^{ST}$. In this case, the regime collapses at $t_0 + k\eta + \tau^{ST}$, and $\tau^{ST*} = \frac{c-g}{g}k\eta = 22.5$, which is irrelevant to $\eta^{LT}$. This explains why all the curves converge to the same flat line when $\tau^{LT}$ is large

enough.

Figure 14 shows how the equilibrium changes in $\eta^{LT}$. From Figure 14 we can see

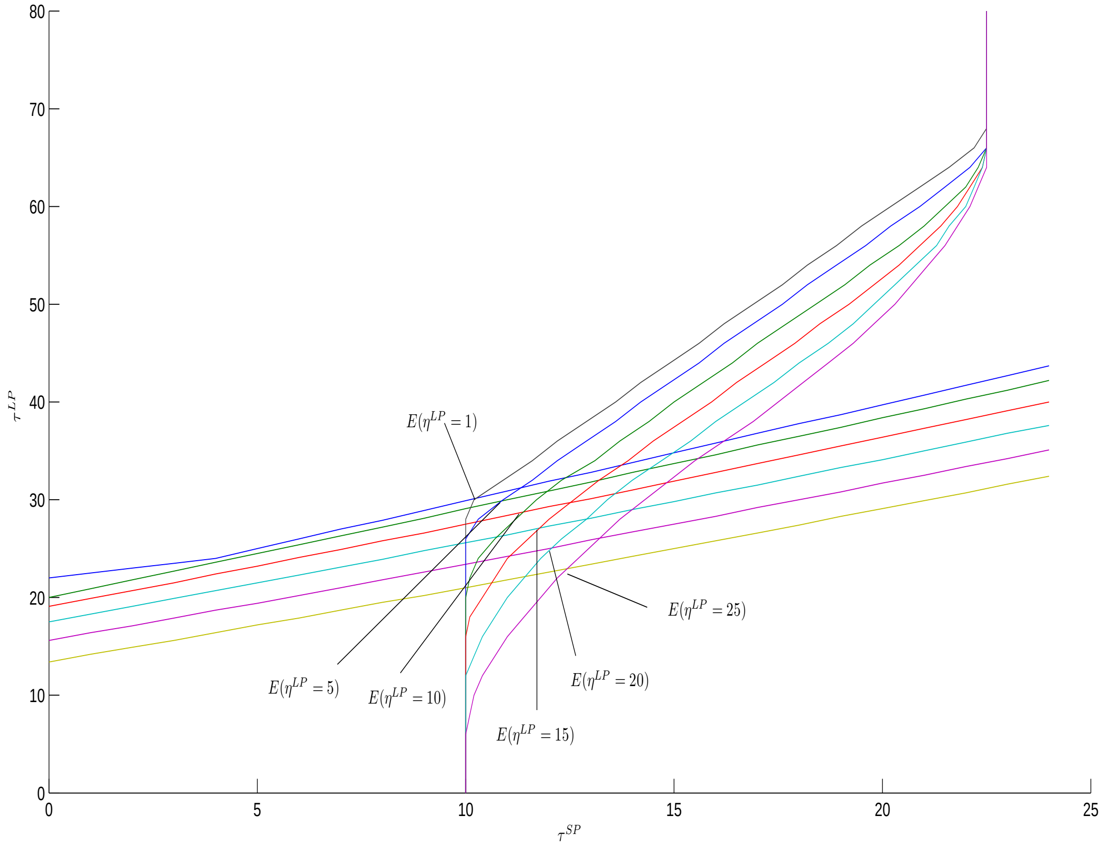


Figure 14: How the Equilibrium Changes in $\eta^{LT}$

that with higher $\eta^{LT}$, the equilibrium strategy of the large trader, $\tau^{LT}$, is smaller, and the equilibrium strategy of small traders, $\tau^{ST}$, is larger. The intuition for this result is straightforward. With large $\eta^{LT}$, the best response of the large trader given the same $\tau^{ST}$, $\tau^{LT}$, is lower, and the best response of small traders given the same $\tau^{LT}$, $\tau^{ST}$, is higher. The co-movement of these two changes leads to the equilibrium change. Note that in Section 4, we study the case when the large trader has perfect information about $t_0$. That is, $\eta^{LT} = 0$. Given the parameter values in our numerical example, $c = 0.015 > g = 0.01$ and $\lambda = 0.5 < \frac{c}{c+g}k = 0.54$. Thus in the equilibrium, $\tau^{LT} = \frac{c}{g}(k - \lambda)\eta = 30$, and $\tau^{ST} = \frac{c-g}{g}(k - \lambda)\eta = 10$. Figure 14 reveals when $\eta^{LT}$ decreases from 25 to 1, the equilibrium does converge to $\tau^{ST} = 10$ and $\tau^{LT} = 30$.

We find that the uncertainty (about the time when the regime collapses) greatly increases when the precision of the large trader's information decreases. In the extreme case where the large trader has perfect information, the time when the regime collapses is perfectly predictable given $t_0$ is known. However, when the large trader has incomplete information about the time when the overvaluation starts, the time when the regime collapses depends crucially on the time when the large trader becomes aware of the overvaluation (how $t^{LT}$ is realized), and we only have a distribution about when the regime will collapse, even given $t_0$ is known. In particular, when the large trader has extremely imprecise information ($\eta^{LT}$ is very large) and a large amount of wealth ($\lambda$ is very large), the presence of a large trader may delay the collapse of the regime ex post, given that the realization of $t^{LT}$ is large. Here we give a numerical example to illustrate this result. We assume that the large trader has the same degree of precision of information as small traders. That is, $\eta^{LT} = 50$. Meanwhile, we keep all the other parameters unchanged. That is, $c = 0.015$, $g = 0.01$, $k = 0.9$, $\lambda = 0.5$, and $\eta = 50$.

Figure 15 shows how the equilibrium changes in $\eta^{LT}$. From the figure we can see that in equilibrium $\tau^{LT}$ is around 11 and $\tau^{ST}$ is around 14. So the regime may collapse at any point between $t_0 + (k - \lambda)\eta + \tau^{ST} = t_0 + 34$ and $t_0 + k\eta + \tau^{ST} = t_0 + 59$. More specifically, with the probability of 46% the regime will collapse at $t_0 + (k - \lambda)\eta + \tau^{ST}$.

With the probability of 4% the regime will collapse at $t_0+k\eta+\tau^{ST}$. With probability of 50% the regime may collapse at any point between $t_0+(k-\lambda)\eta+\tau^{ST}=t_0+34$ and $t_0+k\eta+\tau^{ST}=t_0+59$. Now let us examine the case without a large trader. Assume that the total wealth of both the large and small traders of $1+\lambda$ is now evenly distributed among all the small traders. Then the regime should collapse at $t_0+\frac{k}{1+\lambda}\eta+\tau^{ST}$. Here $\tau^{ST}=\frac{c-g}{g}\frac{k}{1+\lambda}\eta=15$. Therefore, the regime will collapse at $t_0+45$ for sure. Comparing this result with the case with a large trader, we find that

1. With the presence of a large trader, the time when the regime collapses become much more uncertain.

2. Ex post the regime may collapse later in the presence of a large trader. In our example, the regime may collapse between $t_0+45$ and $t_0+59$ with a probability of 32% (when $t^{LT}\in[t_0+34,t_0+50]$). That is, with the probability of 32%, the presence of the large trader will delay the collapse of the regime, instead of accelerating the collapse of the regime.

# 6 Conclusions and Future Research

In this paper we study the role of a large trader in a currency attack using a dynamic currency attack game where traders have to determine when to attack, based on their incentives both to "ride the overvaluation" and to preempt other traders. One of the major results we find is that although the presence of a large trader will always accelerate the collapse of a fixed exchange rate regime when he has perfect information, it may delay the collapse of a fixed exchange rate regime when he has

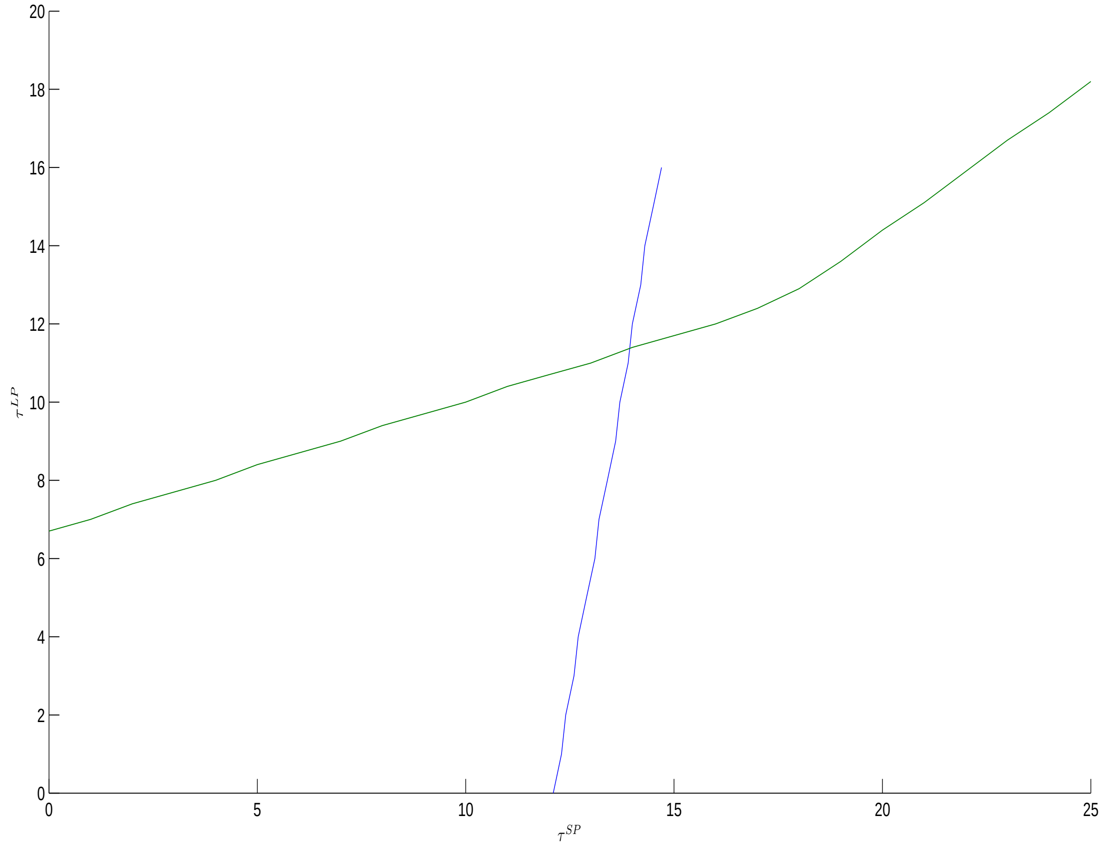


Figure 15: The Equilibrium When $\eta^{LT} = 50$

incomplete information. Moreover, we find that a large trader with a large amount of wealth and very noisy information (but less noisy than that of small traders) can greatly delay the collapse of the regime. Finally, we find that the introduction of a large trader with incomplete information will increase the uncertainty about the time when the regime collapses, compared to the case with only small traders, which demonstrates the difficulty for traders to time the collapse of the regime in reality.

Although our paper is a study on currency attacks, its results can be generalized to all asset markets. Therefore, our paper provides some insight about how a large trader will affect the evolvement of asset bubbles and crashes in general. Our paper demonstrates that the introduction of a large trader to a model with a continuum of small traders can greatly affect the evolution of asset bubbles and crashes. In this sense, our paper reveals that market crashes are very sensitive to the information structure and distribution among agents. Our current results can be interpreted as those attained given that all large traders can collude and act as a single profit-maximizing large trader. In our future research, we plan to generalize our model by introducing multiple large traders who cannot collude and study how the interaction between large traders will affect equilibrium outcomes.

# Appendices

## A

In the benchmark case without a large trader, the regime will collapse at $t_0+\frac{k}{1+\lambda}\eta+\tau^*$, where $\tau^*=\frac{c-g}{g}\frac{k}{1+\lambda}\eta$. Thus, the regime collapses at $t_0+\frac{c}{g}\frac{k}{1+\lambda}\eta$ (here we only consider the case of $c>g$). Recall that in the case with a large trader, the regime collapses at $t_0+\frac{c}{c+g}k\eta$ given $\lambda>\frac{c}{c+g}k$, and at $t_0+\frac{c}{g}(k-\lambda)\eta$ given $\lambda\leq\frac{c}{c+g}k$.

In the case of $\lambda>\frac{c}{c+g}k$, we find that the presence of a large trader will delay the collapse of the regime if and only if $\frac{c}{c+g}k\eta>\frac{c}{g}\frac{k}{1+\lambda}\eta$. Simple algebra shows that it holds if and only if $\lambda>\frac{c}{g}>1$. Since $\lambda<k<1$ by definition, it is not possible. In the case of $\lambda\leq\frac{c}{c+g}k$, we find that the presence of a large trader will delay the collapse of the regime if and only if $\frac{c}{g}(k-\lambda)\eta>\frac{c}{g}\frac{k}{1+\lambda}\eta$. Simple algebra shows that it holds if and only if $k>1$. Since $k<1$ by definition, it is not possible.

## B The Derivation for the First Order Derivative of $E(\Pi^{LT}|t^{LT})$ Given the Parameter Values in Our Numerical Example

Given the parameter values in our numerical example, $t^{LT}-\eta^{LT}+(k-\lambda)\eta+\tau^{ST}<t<t^{LT}+(k-\lambda)\eta+\tau^{ST}$. Or $-\eta^{LT}+(k-\lambda)\eta+\tau^{ST}=-5+\tau^{ST}<\tau^{LT}<20+\tau^{ST}$.

Thus, the first order derivative of $E(\Pi^{LT}|t^{LT})$ w.r.t $t$ gives us:

$$E(\Pi^{LT}|t^{LT}) = \frac{1}{\eta^{LT}}\{\int_{t^{LT}-\eta^{LT}+(k-\lambda)\eta+\tau^{ST}}^{t}(\lambda - \frac{t-s}{\eta})[(k-\lambda)\eta+\tau^{ST}+t-s]gds$$
$$+\int_{t}^{t^{LT}+(k-\lambda)\eta+\tau^{ST}}\lambda\{[(k-\lambda)\eta+\tau^{ST}]g - c(s-t)\}ds\}$$
$$= \frac{1}{\eta^{LT}}\{g\int_{\underline{t}}^{t}[\lambda(k-\lambda)\eta+\lambda\tau^{ST}+\lambda(t-s)-(k-\lambda)(t-s)-\tau^{ST}\frac{t-s}{\eta}-\frac{(t-s)^2}{\eta}]ds$$
$$+\lambda\int_{t}^{\bar{t}}[(k-\lambda)\eta+\tau^{ST}]g - c(s-t)ds\}$$

Let $\lambda(k-\lambda)\eta+\lambda\tau^{ST} = a$ and $2\lambda - k - \frac{\tau^{ST}}{\eta} = b$. Then we have:

$$E(\Pi^{LT}|t^{LT}) = \frac{1}{\eta^{LT}}\{g\int_{\underline{t}}^{t}[a+b(t-s)-\frac{(t-s)^2}{\eta}]ds + \lambda\int_{t}^{\bar{t}}[(k-\lambda)\eta+\tau^{ST}]g - c(s-t)ds\}$$

We have

$$g\int_{\underline{t}}^{t}[a+b(t-s)-\frac{(t-s)^2}{\eta}]ds = g\int_{\underline{t}}^{t}(a+bt-\frac{t^2}{\eta}-bs+\frac{2ts}{\eta}-\frac{s^2}{\eta})ds$$
$$= g[(a+bt-\frac{t^2}{\eta})s|_{\underline{t}}^{t} + (\frac{2t}{\eta}-b)\frac{1}{2}s^2|_{\underline{t}}^{t} - \frac{s^3}{3\eta}|_{\underline{t}}^{t}]$$
$$= g[(t-\underline{t})(a+bt-\frac{t^2}{\eta}) + (\frac{t}{\eta}-\frac{1}{2}b)(t^2-\underline{t}^2) - \frac{t^3-\underline{t}^3}{3\eta}]$$

The first order derivative of the above expression w.r.t $t$ gives us:

$$g[a+b(t-\underline{t})-\frac{(t-\underline{t})^2}{\eta}]$$

Moreover, we have

$$\lambda\int_{t}^{\bar{t}}\{[(k-\lambda)\eta+\tau^{ST}]g - c(s-t)\}ds = \lambda[g(k-\lambda)\eta+\tau^{ST}]s|_{t}^{\bar{t}} + cts|_{t}^{\bar{t}} - c\frac{1}{2}s^2|_{t}^{\bar{t}}$$
$$= \lambda\{[g[(k-\lambda)\eta+\tau^{ST}]+ct](\bar{t}-t) - \frac{1}{2}c(\bar{t}^2-t^2)\}$$

The first order derivative of the above expression w.r.t $t$ gives us:

$$\lambda\{c(\bar{t}-t) - g[(k-\lambda)\eta+\tau^{ST}]\}$$

In sum, the first order derivative of the large trader's expected payoff w.r.t $t$ is given by:

$$g[a + b(t - \underline{t}) - \frac{(t - \underline{t})^2}{\eta}] + \lambda\{c(\bar{t} - t) - g[(k - \lambda)\eta + \tau^{ST}]\}$$

With some constraint on the parameter values, we will find the optimal $t$ that maximizes the large trader's expected payoff.

Reorganize the above expression and we find:

$$\begin{aligned} -\frac{g}{\eta}(t - \underline{t})^2 + (gb - \lambda c)(t - \underline{t}) + ga + \lambda c(\bar{t} - \underline{t}) - g\lambda[(k - \lambda)\eta + \tau^{ST}] \\ = -\frac{g}{\eta}(t - \underline{t})^2 + (2\lambda g - kg - \frac{g}{\eta}\tau^{ST} - \lambda c)(t - \underline{t}) + \lambda c\eta^{LT} \end{aligned}$$

Let $a^* = t - \underline{t}$ denote the optimal solution. Thus we have:

$$\tau^{LT} = a^* - \eta^{LT} + (k - \lambda)\eta + \tau^{ST}$$

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