ANALYSIS OF THE SEEDS OF THE DEBT CRISIS IN EUROPE

Haluk Yener, Thanasis Stengos and M. Ege Yazgan

Department of Economics and Finance,
University of Guelph
DP: 2014-06

Analysis of The Seeds of Debt Crisis in Europe

Haluk Yener[1] Thanasis Stengos[2]
M. Ege Yazgan[3]

September 15th, 2014

Abstract

The paper involves the analysis of the seeds of the recent debt crisis that occurred in the Eurozone area. For the analysis we use the model of Fleming and Stein (2004). This model has two risk drivers arising from uncertainties in the return on capital and the effective rate of return on net foreign assets. Given the risk drivers, we model the net worth value process of an economy under a stochastic setting and show that opening to the rest of the world by pursuing the growth maximizing leverage strategy is better than remaining closed as that strategy enhances the growth of the net worth process. Second, we provide an extra condition to show when the excessive leverage poses threat to the sustainable growth of an economy. This condition allows us to improve upon the predictive ability of the model introduced by Fleming and Stein (2004).

1 Introduction

Current debt trajectories for several economies around the world have been the focus of much attention in recent years as they have been deemed to be unsustainable, something that would have severe repercussions for future growth and stability for these countries. As examples we have Japan which at the end of 2011 had a debt-to-GDP ratio of 233% the highest debt-to-GDP ratio among the world's developed countries, and in Europe, Greece with a 170% debt-to-GDP ratio in 2013 and Italy nearing 130%. The latter two countries alongside Spain and Portugal and Ireland have been at the center of the recent Eurozone debt crisis which has lasted for the last four years. The outlook for a number of countries does not look any better given that the demographic projections for these countries would hinder fiscal spending in the future and make things worse (see Cecchetti et al. (2011a)). The main negative argument rests on the potential crowding out effect of higher public debt on private investment (see Mankiw and Elmendorf (1999)). All this evidence has created an urgent need for policymakers in governments, central banks, and

[1]Department of Business Administration, Istanbul Bilgi University, Santral Campus, 34060, Eyup, Istanbul, TURKEY; haluk.yener@bilgi.edu.tr

[2]Department of Economics, University of Guelph, Guelph, Ontario, N1G 2W1, CANADA; tstengos@uoguelph.ca

[3]Department of Economics, Kadir Has University, 34083, Cibali, Istanbul, TURKEY; ege.yazgan@khas.edu.tr

international policy organizations to understand the effects of debt on economic growth. An additional compounding factor is that investors may interpret the high debt-to-GDP ratios as the result of time inconsistent or inflationary policies which have led countries to implement severe austerity measures on their citizens and adopt fiscal discipline in order to restore their credibility irrespective of the costs in terms of high unemployment, deflation, and the possibility of depression as it has been the case in Greece for the last four years.

In the empirical literature, a recent highly influential study by Reinhart and Rogoff (2010) found that there is generally a weak relationship between government debt and economic growth for countries with debt below 90% of GDP, yet the effect is strongly negative for countries with debt-to-GDP ratio over 90%. Other papers that try to identify nonlinear "threshold effects" of the (public) debt and growth nexus are papers by Cecchetti et al. (2011b) and Caner et al. (2010) who employ the threshold regression of Hansen (2000) to estimate public debt thresholds. Allowing for endogenous threshold effects and the possibility of heterogeneous factors that link public debt to growth Kourtellos et al. (2013) find that the threshold effects are driven by institutions and not debt itself. However, a unifying theme in all the above studies is the preoccupation with public debt and not overall debt, both public and private that an economy faces. Also there is little distinction between internal and external debt holdings. In other words the analysis in the literature fails to address the issue of why a country like Japan does not seem to face the same "treatment by the markets" as a country like Greece. Quality of institutions in a country may give a partial answer to that but it does not seem to be enough to explain the hugely different experience of the two extreme cases mentioned above.

In this paper we conduct the analysis of the recent debt crisis that occurred in the Eurozone area using a different framework allowing for the total overall external debt to be taken into account. We consider eight countries and divide them into two groups that are categorized as resilient and non-resilient. We then use the model of Fleming and Stein (2004) as an early warning signal of the debt crisis and provide an extra condition that strengthens the prediction ability of the signal. This condition translates to no admissible leverage strategy for turning the economy around during a downturn if the fundamentals are not favourable enough. This allows us to examine precisely how excessive risk taking due to over-borrowing is a danger to an economy.

The model of Fleming and Stein (2004) consists of a stochastic net worth process that is the result of the difference between the capital and debt stock in an economy. Its stochasticity arises from two correlated Brownian motions that are used for modeling the uncertainties arising from the return on capital and the effective rate of return on net foreign assets. By involving these motions, Fleming and Stein (2004) creates a dynamic model and apply Merton's approach (see Merton (1971) and Merton (1990))[4]. Thanks

[4]This approach involves the specification of an objective function in the form of maximizing or minimizing a utility function from consumption and/or terminal wealth subject to a continuously traded portfolio process in frictionless markets. The portfolio process is in turn created via investment into a

to the similarity of their model to a stochastically traded portfolio process, Fleming and Stein (2004) used Merton's approach to find the optimal leverage strategy based upon the observed state of the net worth process. To this end, they used a hyperbolic absolute risk aversion (HARA) type utility function and derived the optimal leverage and consumption rate of an economy. They later showed how their results are related to the mean-variance approach and how their analysis can be used to detect possible debt crises. To realize the latter, Fleming and Stein (2004) derive an optimal leverage strategy and use it as a benchmark against the actual leverage. If the actual leverage rises above the benchmark significantly, it is then assumed to become unsustainable and a possible debt crisis is expected to occur With this claim in mind, Fleming and Stein (2004) show how their model correctly predicts the agricultural debt crisis that took place in the first half of the 1980s in the US.

In an earlier paper Fleming and Stein (2001) used stochastic optimal control theory techniques in discrete time and with a finite horizon in order to determine the optimal foreign debt level. Mainly, by assuming an uncertain productivity rate and imposing a no default constraint they used the optimal debt level as a benchmark and established that the deviation of the actual debt from the benchmark beyond a maximal debt level signals debt crises. In other words, if exposure to debt is beyond a certain level when compared to the level given by the fundamentals, a debt crisis is expected to happen. Using the above framework, Stein and Paladino (2001) conducted an empirical assessment of country default risk in emerging markets by considering a group of twenty-one countries that defaulted on the debt during the period 1978-99 and a control group of thirteen countries that did not default. Through this assessment they showed how the model can be used as an early warning signal for default prediction.

Some other relevant studies include Stein (2005) on the analysis of the debt problems due to Asian crisis during 1997-1998 and the Argentinian crisis in 2001, Stein (2007) on the analysis of the US current account deficit, Stein (2010) on US Agricultural debt crisis during 1980s and the subprime mortgage crisis in 2007, and Stein (2009) on the application of the model to financial market debt crisis when the uncertainty is due to changes in the price level instead of the interest and productivity rates. In sum, the original approach of Stein and Fleming eventually evolved into various models that can be used for debt crisis prediction[5] of a group of countries, or a country, or even a sector in a country. Therefore, given the empirical support, a suitable goal would be then using their model for the analysis of recent debt crisis in Europe.

A previous study regarding the crisis was done by Stein (2011), where Stein showed the underlying differences in the causes of the debt crises across Europe. While the crises in Ireland and Spain occurred mainly due to the private sector borrowing, particu-

riskless and $N \geq 1$ risky assets. Given the structure of the portfolio, the aim of the investor is to find the optimal investment strategy that will lead to the realization of her objective.

[5]We refer the readers to Stein (2006) and Stein (2012) that offer the collection of the studies in the field.

larly housing, in Greece and Portugal they occured due to cyclically adjusted structural deficits. Hence, Stein suggested to use as an early warning signal the difference between the actual leverage and optimal leverage ratios. However, rather then using data from national accounts for his analysis, Stein interpreted and used various data from secondary sources related to interest rates, gross domestic product, government structural balance, government net debt, residential property prices, rent prices, and return on assets along with the optimal leverage strategy specification in order to reach the conclusions stated in the paper. Our approach differs from his in that we use the national account data for our analysis and instead of differentiating between private and public debt as the root cause of the Eurozone crisis, we show how excessive risk taking along with deteriorating fundamentals led to the crisis.

Based on the model of Fleming and Stein (2004), an economy will take the extra risk arising from the effective rate of return on its net foreign assets if it decides to remain open to the rest of the world. That is, leverage is used if the economy decides to finance itself (provide funds) from (to) external resources. Otherwise, the risk in an economy will only be due to the uncertainty in return on capital. Then, by intuition, we may say that additional risk will be taken if leverage brings benefits in terms of enhancing the growth of the net worth of an economy. Our results show that by pursing the growth maximizing leverage strategy, which is the strategy that maximizes the trend of the net worth process, an economy will be better off in opening to the rest of the world in terms of improving the growth of its net worth. Furthermore, using arguments from the portfolio optimization literature[6], we show the high levels of risk that exist when pursuing the growth maximizing strategy and define, from the risk and security tradeoff argument in MacLean et al. (2011), the growth maximizing strategy as the limiting case.

Our main goal with defining the growth maximizing strategy as the limiting case is to show that if an economy's actual leverage strategy is higher than the level denoted by the growth maximizing strategy then that economy takes excessive risk by giving up growth. That is, if the difference between the actual leverage strategy and the growth maximizing strategy is positive, then an economy is exposed to high levels of risk. However, although an economy may take excessive debt, if its fundamentals are strong enough, it may be able to adjust to a new equilibrium without default in case of a downturn. To measure the strength of the fundamentals, we use the argument provided in Browne (1999)[7], which

[6]We refer the readers to the study of MacLean et al. (2010) on a discussion regarding the good and bad properties of the growth maximizing strategy.

[7]Browne considered an investor who continuously trades a portfolio by splitting her funds between risky assets that are named as stocks and a risk-free asset that grows with a constant risk-free interest rate. He showed that the growth maximizing strategy both maximizes the expected time a traded portfolio process stays above a benchmark (also known as the survival time) and minimizes the expected time a traded portfolio beats a benchmark process (also known as the goal reaching time). Furthermore, he also showed that if the trend of the portfolio traded under the growth maximizing regime is negative (which he defines as the unfavourable market) then there is no admissible trading strategy for which the goal reaching time is finite. On the other hand, if the trend of the portfolio traded under the growth

translates to the existence of no admissible leverage strategy for turning the economy around during a downturn if the fundamentals are not favourable enough. That allows us to show that excessive risk taking by an economy when its fundamentals are not strong leads to debt problems.

To find the results we first provide the framework in section 2. There, we construct the model in subsection 2.1 and provide the method in subsection 2.2. For the construction, we consider infinite horizon and assume that the consumption is proportional to the net worth value. Then, we proceed to section 3 for the description of the data used for the analysis and conduct the numerical analysis of the candidate economies for our studies. We conclude by summarizing the findings in section 4.

2 The Framework

2.1 The Model

We start by providing the mathematical preliminaries for modeling the uncertainty in our economy. Note that we consider infinite time horizon. Furthermore, we assume that the economy is modeled under a complete filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{0\leq t<\infty}, \mathbb{P})$ where $\mathbb{P}$ denotes the real measure, and $\{\mathcal{F}_t\}_{0\leq t<\infty}$ is the $\mathbb{P}$-augmentation of the natural filtration $\mathcal{F}_t^W := \sigma\{W(u) \mid u \leq t\}$. Here, $W(\cdot)$ is the 2-dimensional Brownian motion $W(t) := (W_1(t), W_2(t))^{'}, 0 \leq t < \infty$ defined on our complete probability space. Via the filtration $\{\mathcal{F}_t\}_{0\leq t<\infty}$ we model the evolution of the uncertainties in an economy. This is equivalent to finer partition of Ω, which is the set of all possible events in that economy. On the other hand, the sigma algebra $\mathcal{F}_t$, which is the set of all subsets of Ω, provides information about the status of the economy.

We assume that the Brownian motions $W_1(\cdot), W_2(\cdot)$ are correlated. That is,

$$dW_1(t)dW_2(t) = \rho dt \tag{2.1}$$

where $|\rho| \leq 1$ denotes the correlation coefficient and is a constant.

The Brownian motions are used to model the uncertainties in the economy. We assume that the uncertainties arise from the effective rate of return on debt and the return on capital. We formulate these processes respectively by

$$r(t)dt = r_t dt + \sigma_1 dW_1(t); \tag{2.2}$$

$$b(t)dt = b_t dt + \sigma_2 dW_2(t), \tag{2.3}$$

where $b_t > 0$ is the deterministic mean rate of change in the return on capital and $r_t > 0$ is the deterministic mean effective rate of return on debt. We will define what we mean

maximizing regime is positive (this is defined to be the favourable market case), then there will always be an admissible strategy for which the survival time is infinite.

by debt in the following parts of this section. In addition, we assume that the standard deviations of our processes, $\sigma_1 > 0, \sigma_2 > 0$, are constants.

We start constructing the model by first defining the change in the capital stock of the economy at time t. To this end, we let $N(t)$ be the number of units of capital at time t, $P(t)$ be the unit price of capital at time t, and $K(t) > 0$ be the capital stock at time t. Note that, as explained in Stein (2007), $N(t)$ denotes the quantity while $P(t)$ denotes the quality. Then, the capital stock at time t is given by

$$K(t) = N(t)P(t). \tag{2.4}$$

We also let $I(t)$ be the investment rate. From (2.4), the change in the capital stock is written as

$$dK(t) = N(t)dP(t) + P(t)dN(t) = K(t)\frac{dP(t)}{P(t)} + I(t)dt, \tag{2.5}$$

where $\{dP(t)/P(t), t \geq 0\}$ is the stochastic capital gain. Furthermore, we define the gross domestic product, $Y(t)$, as

$$Y(t)dt = \alpha(t)K(t)dt, \tag{2.6}$$

showing that $Y(t)$ is the product of the stochastic productivity rate, $\{\alpha(t), t \geq 0\}$, and the capital stock. Next, we let $L(t)$ and $C(t) > 0$ denote respectively the debt value and the consumption rate at time t. The balance equation for the change in debt is

$$dL(t) = [r(t)L(t) + I(t) + C(t) - Y(t)]\, dt. \tag{2.7}$$

As indicated in Fleming and Stein (2004), the above equation is equivalent to the current account deficit. In this regard, $L(t)$ denotes the value of the net foreign assets of an economy; it is the sum of the net transfer payments $r(t)L(t)$ and the trade balance $I(t) + C(t) - Y(t)$. Here, $r(t)$, the net effective rate of return, contains the interest and dividends on the net foreign assets. Notice that the case $L(t) > 0$ denotes the case of a debtor, while $L(t) < 0$ denotes case of a creditor economy.

On the other hand, the consumption is given proportionally by $C(t, X(t)) = c_t X(t)$, where $c_t \geq 0$ is the deterministic rate of consumption and $X(t)$ is the net worth process given by

$$X(t) = K(t) - L(t), \tag{2.8}$$

which is constrained to have $X(t) > 0$. In other words, this constraint guarantees that a Ponzi scheme is not possible. That is, debt is not refinanced by further borrowing.

Now, given, the deterministic b_t, r_t, c_t, we assume that

$$\int_0^t (|r_s| + |b_s| + |c_s|)\, ds < \infty \quad \text{for } t < \infty \tag{2.9}$$

With the above assumption and with what follows, our aim is to provide the evolutionary dynamics of the net worth process.

Next, by using equations (2.2)-(2.8) we express the change in the net worth by

$$
\begin{aligned}
dX(t) &= dK(t) - dL(t) \\
&= \left[\alpha(t) + \frac{dP(t)}{P(t)}\right] K(t) - C(t)dt - r_t L(t)dt - \sigma_1 L(t) dW_2(t). \qquad (2.10)
\end{aligned}
$$

From the stochastic part $\{\alpha(t) + dP(t)/P(t), t \geq 0\}$, we observe that growth in productivity and capital gain leads to an increase in the net worth of an economy. However, for a given level of capital gain and technical progress, an increase in consumption causes a decline in the net worth, since from equation (2.7), we observe that a growth in consumption leads an increase in debt driving the net worth towards zero. On the other hand, an increase in $I(t)$ leads to an increase both in the capital stock and the net foreign assets as we may observe from specifications (2.5) and (2.7) respectively. Therefore, any foreign borrowing directed towards investment may end up increasing the net worth of an economy. In sum, the model captures the benefit of investment to an economy while it shows the cost of excessive consumption.

We model the stochastic part $\{\alpha(t)+dP(t)/P(t), t \geq 0\}$ by using the stochastic return on capital, $\{b(t), t \geq 0\}$, given in equation (2.3). Therefore, by using (2.8) as well, we write (2.10) as

$$
dX(t) = [b_t X(t) + (b_t - r_t)L(t) - C(t)]\, dt - L(t)\sigma_1 dW_1(t) + (X(t) + L(t))\sigma_2 dW_2(t). \qquad (2.11)
$$

Notice that $I(t)$ is not present in the stochastic differential equation above. This equation can also be expressed via control process $\mathbf{f}(t) = L(t)/X(t)$ named as the leverage strategy. By using (2.8) we also find $\mathbf{k}(t) = K(t)/X(t) = 1 + \mathbf{f}(t)$, the fractional capital amount. Therefore, controlling the leverage strategy is equivalent to controlling the capital strategy, and thus, the rate of investment. Therefore, from the identity $\mathbf{k}(t) = 1 + \mathbf{f}(t)$, we see that borrowing is directed towards capital investment which may in turn lead to an improvement in the net worth of an economy.

We say that the leverage strategy is admissible for an initial net worth amount x, that is $\mathbf{f} \in \mathcal{A}(x)$, if $\mathbf{f}(t)$ is $\{\mathcal{F}_t\}$-progressively measurable and satisfies $\int_0^t \mathbf{f}^2(s)ds < \infty$ for $t < \infty$. In words, leverage level must depend on the observed value of the net worth process and its value must be finite since resources are scarce. Then, the net worth process associated to an admissible leverage process $\mathbf{f}(t)$ is the solution of the stochastic differential equation

$$
\begin{aligned}
dX^f(t) &= [(b_t - c_t) + (b_t - r_t)\mathbf{f}(t)]\, X^f(t)dt - \mathbf{f}(t)X^f(t)\sigma_1 dW_1(t) \\
&\quad + (1 + \mathbf{f}(t))X^f(t)\sigma_2 dW_2(t); \\
X(0) &= x. \qquad (2.12)
\end{aligned}
$$

With the above specification we guarantee that a Ponzi scheme is not possible since $X^f(t) > 0$ for $t < \infty$. In addition, we assume no frictions. Therefore, the leverage strategy can be changed instantaneously and costlessly.

2.2 Growth Maximization

Without exposure to debt the closed form solution of the net worth process is

$$X(t) = x \exp\left\{ \int_0^t \left[b_s - c_s - \frac{1}{2}\sigma_2^2 \right] ds + \sigma_2^2 W_2(t) \right\}. \tag{2.13}$$

We obtain the above equation by setting $\mathbf{f}(t) = 0$ in equation (2.12) and applying Itô's formula[8]. The specification in equation (2.13) is for an economy that finances its consumption from its own internal resources. This is the case of a closed economy. However, by opening to external sources, an economy might find a way to improve its growth. In other words, if it finds a leverage strategy that adds to the instantaneous trend in (2.13) defined by

$$b_t - c_t - \frac{1}{2}\sigma_2^2,$$

then, it can enhance growth with leverage. Therefore, in this section, we consider a growth problem related to finding an optimal leverage strategy that maximizes the net worth of an economy by improving its trend. As shown in Davis (2004), in order to find such a strategy we write by the application of Itô's formula to $\log(X^f(t))$, the stochastic differential equation

$$\begin{aligned} d(\log(X^f(t))) &= [(b_t - c_t) + (b_t - r_t)\mathbf{f}(t) - \frac{1}{2}\mathbf{f}^2(t)\sigma_1^2 + (1+\mathbf{f}(t))^2\sigma_2^2 \\ &\quad -2\mathbf{f}(t)(1+\mathbf{f}(t))\rho\sigma_1\sigma_2]dt - \mathbf{f}(t)\sigma_1 dW_1(t) + (1+\mathbf{f}(t))\sigma_2 dW_2(t). \end{aligned} \tag{2.14}$$

The leverage strategy that maximizes the trend can then be found by

$$\mathbf{f}^* := \arg\sup_{\mathbf{f}} [(b_t - c_t) + (b_t - r_t)\mathbf{f} - \frac{1}{2}\mathbf{f}^2\sigma_1^2 + (1+\mathbf{f})^2\sigma_2^2 - 2\mathbf{f}(1+\mathbf{f})\rho\sigma_1\sigma_2].$$

By solving the above problem, we find the specification of the growth maximizing leverage strategy as

$$\mathbf{f}^*(t) = \frac{b_t - r_t}{\sigma^2} + \alpha(\rho\kappa - 1), \tag{2.15}$$

where $\sigma^2 = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2$, $\alpha = \sigma_2^2/\sigma^2$, and $\kappa = \sigma_1/\sigma_2$. We observe that the growth maximizing strategy is independent of the consumption rate. Thus, an economy seeking to maximize its growth pursues a leverage strategy that is only dependent on the the parameters concerning the mean rate of return on its capital and its effective rate of return on its net foreign assets. Next, we observe that substituting (2.15) into (2.12) and

[8]See Shreve (2004) for example on the application of this formula.

solving the stochastic differential equation gives, for $t < \infty$, the optimal net worth process as

$$\begin{aligned} X^*(t) &= x \exp\left\{ \int_0^t \beta_s ds - \int_0^t \left(\frac{b_s - r_s}{\sigma^2} + \alpha(\rho\kappa - 1) \right) \sigma_1 dW_1(s) \right. \\ &\quad \left. + \int_0^t \left(1 + \frac{b_s - r_s}{\sigma^2} + \alpha(\rho\kappa - 1) \right) \sigma_2 dW_2(s) \right\}, \end{aligned} \tag{2.16}$$

where β_s, the trend of the net worth process, is given by

$$\beta_s = (b_s - c_s) - \frac{1}{2}\sigma_2^2 + \frac{1}{2}\left[\frac{b_s - r_s}{\sigma} + \sigma(\alpha(\rho\kappa - 1)) \right]^2. \tag{2.17}$$

Therefore, with the growth maximizing strategy, the trend of the net worth process, as it can be seen in (2.16), becomes equal to β_t. From this, we observe that if an economy becomes open by following the growth maximizing strategy, it will be able to improve its growth. The value of improvement is, in turn, equal to

$$\frac{1}{2}\left[\frac{b_s - r_s}{\sigma} + \sigma(\alpha(\rho\kappa - 1)) \right]^2 > 0.$$

However, while exposure to debt in the aforementioned form can maximize the growth, it also brings a potential downside risk threatening the growth sustainability of an economy. Especially, in the portfolio optimization literature, while growth maximizing strategy is deemed to bring largest wealth levels, it may also bring large losses because an investor commits large wagers making an investment strategy prone to big losses. Therefore, there is a risk and security tradeoff (See, MacLean et al. (2011)). In other words, pursuing a fractional growth maximizing strategy reduces the possibility of big losses. To see how, consider the maximization of the power type utility function given by $U(x) = x^{1+p}/1 + p$ where $p \in (-\infty, 0) \setminus -1$ is the risk aversion parameter. In this case, the optimal leverage strategy would be (see Fleming and Stein (2004))

$$\mathbf{f}^*(x) = -\frac{1}{p}\frac{b - r}{\sigma^2} + \alpha(\rho\kappa - 1). \tag{2.18}$$

As we observe from (2.18) the leverage strategy is dependent on the level of the risk aversion parameter p. Here, while $p \in (-\infty, -1)$ denotes the case of a risk averse economy, $p \in (-1, 0)$ denotes the case of a risk loving one. Note that for $p = -1$ we obtain the growth maximizing strategy and in this case the utility function becomes a logarithmic one. Therefore, as long as an economy is risk averse, its optimal leverage strategy will be in fractional growth maximizing amount.

The case with $p \in (-1, 0)$ on the other hand corresponds to the levels of leverage much riskier than the level an economy would obtain with the growth maximizing strategy. In fact, for $p \in (-1, 0)$ an economy would not only take high risk but also grow less. That is, if an economy's actual leverage strategy is the one obtained under the case when $p \in (-1, 0)$, then such economy not only gives up growth, but also takes excessive debt that will put the sustainability of its growth at a risk level that is even higher than the one denoted by the growth maximizing strategy. For this reason, the growth maximizing leverage strategy may be considered as the limiting case and any riskier leverage strategy may be a warning signal (let alone the case with the growth maximizing one) for possible debt crisis.

However, violation of the limiting case may not be solely enough to anticipate a debt crisis as the fundamentals of an economy may lead to a new equilibrium without default. That is, while an economy may take excessive risk by borrowing more than the level denoted by the growth maximizing strategy, it may not default on the debt if its fundamentals are strong enough for a smooth adjustment. To see how this may be so, we refer to the arguments in Browne (1999). To this end, we consider two barrier points U and L such that $L < x < U$, and let

$$\tau_U^f = \inf\{t > 0 \mid X^f(t) \geq U\}$$

be the first time the net worth process crosses the higher net worth level under a leverage strategy $\mathbf{f}$, and

$$\tau_L^f = \inf\{t > 0 \mid X^f(t) \leq L\}$$

be the first time the net worth process crosses the lower net worth level under a leverage strategy $\mathbf{f}$. Then, we define for $\mathbb{E}_x[\cdot] = \mathbb{E}[\cdot \mid X(0) = x]$, $\mathbb{E}_x\left[\tau_L^f\right]$ as the expected time the net worth process hits the lower barrier L and $\mathbb{E}_x\left[\tau_U^f\right]$ as the expected time the net worth process hits the upper barrier level U. In short, we call the former the survival time and the latter the goal reaching time.

Then, by applying the analysis of Browne (1999) to our case, we have for $\beta_t < 0$ (which is the equivalent of the unfavourable markets defined in Browne (1999)), $\inf_{\mathbf{f}} \mathbb{E}_x[\tau_U^f] = \infty$. That is, as long as an economy stays in a bad condition, there is no admissible leverage strategy that will improve the net worth of an economy toward a higher target. Reaching such a target is only possible with improved fundamentals. In other words, whenever the factors of the economy give $\beta_t > 0$ (which is the equivalent of the favourable markets defined in Browne (1999)), then, $\sup_{\mathbf{f}} \mathbb{E}_x[\tau_L^f] = \infty$. That is, when the condition of the economy becomes good, there will always be an admissible leverage strategy that will prevent the economy from deterioration.

In sum, as long as structural problems are persistent, an economy can only maximize the expected survival time by pursuing the growth maximizing strategy. However, it cannot manage to change the course of its net worth toward a higher target level with the same strategy. Changing the course of an economy is only possible via structural change. If there is no change, then the best an economy can do it to maximize its survival time.

Therefore, pursuing a leverage strategy that is even riskier than the one denoted by the growth maximizing strategy leads to a faster deterioration which may in turn lead to a debt crisis. As a result, the case when $\beta_t < 0$ and $\mathbf{f}(t) - \mathbf{f}^*(t) > 0$, where $\mathbf{f}(t)$ is the actual leverage strategy at time t, is the warning sign for an economy. In the next section, we name the warning sign as the non-resilience condition and show that its persistence when coupled with excessive debt leads eventually to a debt crisis.

3 The Data & Analysis

3.1 The Data & Estimation Procedures

The stochastic net worth process considered in the paper contains five key variables, which are mainly capital stock, debt stock, investments, gross domestic product and consumption. All data for these variables are denominated in US dollars and retrieved for the period between years 1980 and 2007 on annual basis. The capital stock data are obtained from Fred Economic Data website that contains the estimations of Feenstra and Timmer (2013). Investments, income and consumption data are on the other hand retrieved from World Bank's World Development Indicators database. For investments and consumption, we use gross capital formation and final consumption expenditure data respectively.

As we mentioned previously, the debt is the net foreign assets of an economy. The data for this variable is obtained from the updated and extended version of the External Wealth of Nations Mark II database developed by Lane and Milesi-Ferretti (2007). We use the data both to compute the net worth of an economy and to estimate the effective rate of return, $r(t)$, on net foreign assets. Since the data are given on annual basis, the estimation of $r(t)$ is done via the discretized version of the closed form solution to (2.7). That is, we solve for

$$r(t) = \frac{\Delta L(t) - C(t+1) - I(t+1) + Y(t+1)}{L(t)},$$

where $\Delta L(t) = L(t+1) - L(t)$. Under classical assumption, the change in the net foreign assets is given by the current account deficit (See for example Stein (2007) when calculating the US net foreign assets). However, in reality the change must also account for the valuation effects as the change in the exchange rates and changes in the prices of assets and liabilities have an impact on the value of the net foreign assets of a country. Since the data of all nations considered in this study are expressed in US dollars, then we calculate $r(t)$ as above (rather than simply computing it via interest rates) to reflect the effect of the valuation changes.

Furthermore, another key parameter, $b(t)$ is estimated as in Stein (2007) by setting $b(t) = \Delta Y(t)/\Delta K(t)$, giving us the income generated from a dollar increase in the capital stock. We compute the deterministic parameters r_t, b_t and c_t by taking the three year moving averages of the relevant data. Once the necessary data are obtained, we then compute the growth maximizing strategy $\mathbf{f}^*$ along with the actual strategy $\mathbf{f}$, and β for a group of eight countries to study the seeds of the recent debt crisis in Europe.

The list of countries is divided into two groups based on their resilience:

- The resilient nations are: Austria, Belgium, Germany and France;
- The non-resilient nations are: Greece, Italy, Portugal and Spain.

We present in the next two graphs the actual leverage values of the two groups. We observe in figure 1 that, except Austria, all nations are creditors to the rest of the world. However, Belgium and France became creditors around 1992 and 1997 respectively, while Germany was creditor for the whole period.

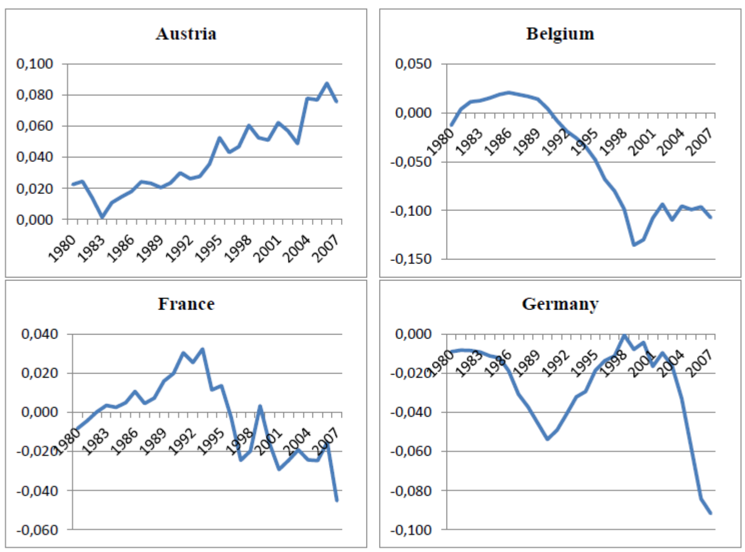

Fig. 1: Actual Leverage of Resilient Group

On the other hand, in figure 2, we observe that all nations were debtors for the whole period. Especially, after 2000 the leverage levels increased to much higher values relative to the historical record before year 2000. Mainly, the leverage level, while it was lower in Italy relative to other nations, it increased in Greece, Portugal and Spain to the order of around 40% to 60%. Finally, observe in both graphs that the leverage levels of both Austria and Italy came up to same levels. However, while one nation is in the resilient group the other is not. We will see why this is so in the numerical analysis section.

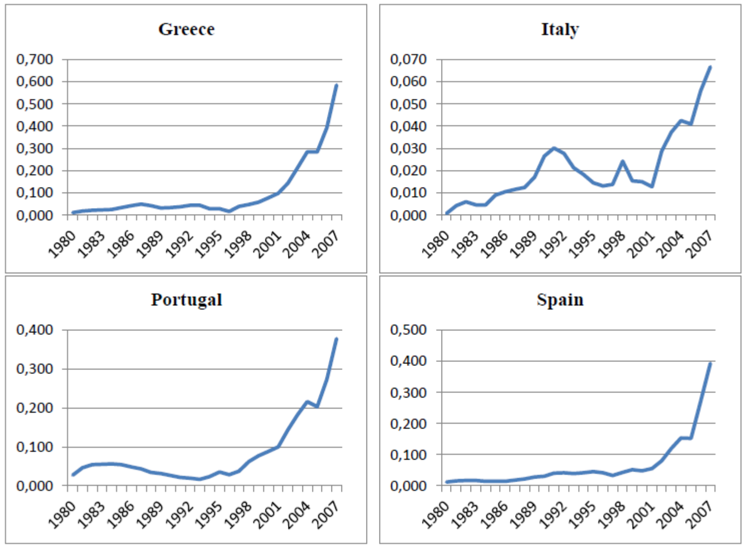

Fig. 2: Actual Leverage of Non-Resilient Group

3.2 The Numerical Analysis

In this section, we provide the analysis of the results we obtained for the nations considered in this paper. In figures 3 and 4, we provide the graphs that depict the time series of β and $\mathbf{f} - \mathbf{f}^*$ for the period between 1982 and 2007. The period starts in 1982 as the variables b_t, r_t and c_t are obtained by taking the three year moving averages of the data starting in 1980.

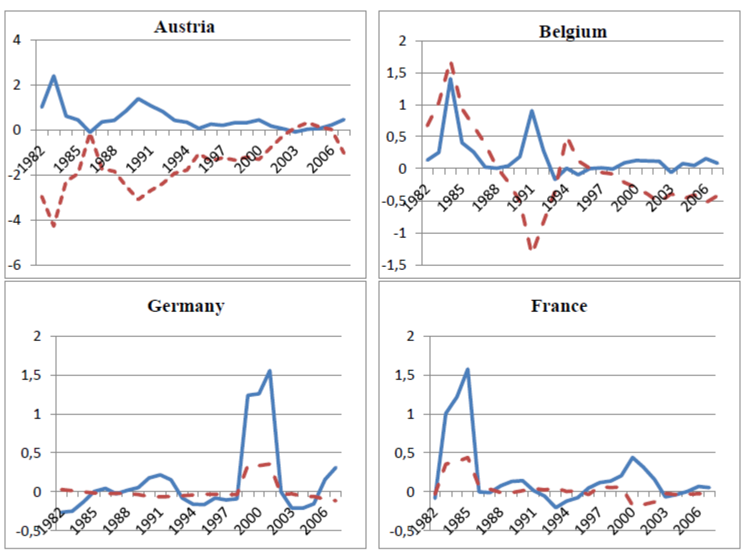

Fig. 3: β vs. ***f-f****** of the Resilient Group.
The solid line represents β and the dashed line represents ***f-f****.

We first provide the analysis of the resilient nations. As we observe from figure 3, the condition of non-resilience, which is $\beta < 0$ and $\mathbf{f} - \mathbf{f}^* > 0$, is satisfied for Belgium in 1995 and 1996. However, from figure 1, we already saw that Belgium became a creditor nation around 1992. For this reason, the condition of non-resilience cannot be interpreted as an early warning signal. The same is true for Germany during the period 1982-1985.

On the other hand, for Austria, we observe that $\beta > 0$ and $\mathbf{f} - \mathbf{f}^* < 0$ for the most period except in 2003. Therefore, 2003 is the only time when Australia satisfies the non-resilience condition. However, the non-resilience period was short-lived and the growth picked up again with improving fundamentals that eventually led to positive β and negative $\mathbf{f} - \mathbf{f}^*$. Moreover, as a debtor nation until 1996, France satisfied the condition of non-resilience during the period 1992-1995. However, France's had low realized leverage (between 1.2% and 3.2%) during the period and became a creditor nation after 1995. In short, we see from the analysis of both nations that when the non-resilience condition is not persistent and the realized leverage is low, then a nation will have the chance to avoid a possible debt crisis.

The case of non-resilient nations is different from France and Austria in that these nations have an increasing debt trend complemented with persistent non-resilience condition during the analysis period and with an ongoing deterioration that eventually leads to non-resilience. For example, from figure 4, we see that while Portugal's fundamentals would allow for increasingly positive β with declining negative $\mathbf{f} - \mathbf{f}^*$ between the period 1986-1994, from 1994 β started to deteriorate along with increasing $\mathbf{f} - \mathbf{f}^*$ that eventually followed with persistently negative β and positive $\mathbf{f} - \mathbf{f}^*$ after 2002. If 1994 was the year when the deterioration in the fundamentals started, as we may observe from figure 2, it was also the year when the realized leverage of Portugal started to increase. Increasing realized leverage along with persistent non-resilience condition are the main factors for Portugal's debt problems.

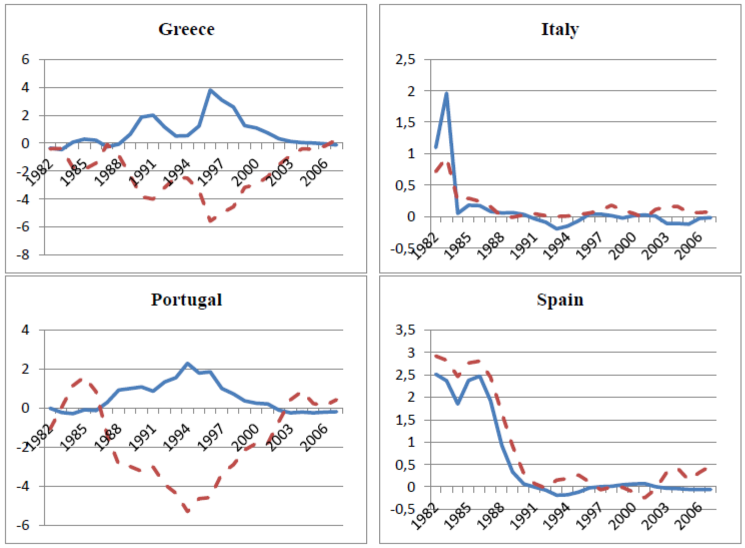

Fig. 4: *β* vs. ***f-f****** of the Non-Resilient Group.
The solid line represents *β* and the dashed line represents ***f-f*****.

The case with Greece is also similar to that of Portugal. As we observe from figure 2, there was a consistent decline in β and an increase in $\mathbf{f}-\mathbf{f}^*$ after 1997. This was the year when the realized leverage of Greece started to increase as well. Increasing realized leverage to around 60% of the net worth along with deteriorating fundamentals leading to negative β and positive $\mathbf{f}-\mathbf{f}^*$ after year 2006 is a clear signal for the Greece's recent debt crisis. As for Spain on the other hand, we observe that both β and $\mathbf{f}-\mathbf{f}^*$ started to decline toward zero from positive values during the period between 1982 and 1991. After 1991 both indicators fluctuated around zero till 2002. After that year, we see the persistence of the non-resilience condition along with increasing realized leverage to around 40% of the nation's net worth. As a result, the deteriorating fundamentals coupled with excessive leverage caused the debt crisis of Spain.

Finally, from figure 4, we see that β and $\mathbf{f}-\mathbf{f}^*$ of Italy fluctuates around zero since 1984. However, after 2002, we see that β remained negative and $\mathbf{f}-\mathbf{f}^*$ remained positive consistently. As for the actual leverage, we observe from figure 2 that it increased from around 2.9% in 2002 to 6.7% in 2007. Here as well non-resilience is coupled with excessive leverage, but, the realized leverage levels seen in Italy are closer to those of Austria, which is considered resilient in this study. Nevertheless, a comparison two nations from figures 3 and 4, yields that Italy had much worse performance in terms of the values of β and $\mathbf{f}-\mathbf{f}^*$ during the analysis period. Therefore, even if Italy had the lowest leverage levels among the non-resilient nations, its fundamentals were not strong enough to save the country from its recent debt problems.

4 Conclusion

In this paper, we used the model of Fleming and Stein (2004) to study the seeds of the recent debt crisis in Europe. To this end, we used the growth maximizing strategy as the benchmark in order to compare it with the realized leverage strategy of an economy. Our main idea behind using the growth maximizing strategy was to show that this strategy enhances the growth of the net worth of an economy. That is, opening to the rest of the world under this strategy is better than remaining closed. However, while this strategy enhances growth it may also bring large losses as it involves high risk. On the other hand, given the risk and security tradeoff with the fractional growth maximizing strategy and the excessive risk, lower growth possibility with the multiples of the growth maximizing strategy, we then define the growth maximizing strategy as the limiting case and use it as a benchmark to detect signals of a possible debt crisis. However, exceeding the growth maximizing strategy may not solely be enough as an early warning signal as the fundamentals of an economy may be strong enough for a smooth adjustment. We show a method to measure the strength of the fundamentals by using the arguments in Browne (1999) and create a non-resilience condition. Then, we show by applying to a group of eight countries that if the non-resilience condition is associated with prolonged and persistent deterioration in fundamentals and increasing excessive leverage, it then signals a debt crisis.

References

Browne, S., 1999. Beating a moving target: Optimal portfolio strategies for outperforming a stochastic benchmark. Finance and Stochastics 3, 275–294.

Caner, M., Grennes, T., Koehler-Geib, F., 2010. Finding the tipping point when sovereign debt turns bad. Policy Research Working Paper Series 5391, The World Bank.

Cecchetti, S., Mohanty, M., Zampolli, F., 2011a. Challenges to central banking in the context of the financial crisis. Academic Foundation: New Delhi, Ch. The future of public debt, pp. 183–217.

Cecchetti, S., Stephen, G., Mohanty, M., Zampolli, F., 2011b. The real effects of debt. Working paper, Bank for International Settlements (BIS).

Davis, M. H. A., 2004. Applied Mathematics Entering the 21st Century. Society for Industrial and Applied Mathematics, Philadelphia, PA, Ch. Valuation, hedging and investment in incomplete financial markets, pp. 49–70.

Feenstra, Robert C., R. I., Timmer, M. P., 2013. The next generation of the penn world table. Available for download at www.ggdc.net/pwt.

Fleming, W. H., Stein, J. L., 2001. Stochastic inter-temporal optimization in discrete time. In: Takashi, N., Ramachandran, R., Mino, K. (Eds.), Economic Theory Dynamics and Markets: Essays in Honor of Ryuzo Sato. Kluwer, Boston.

Fleming, W. H., Stein, J. L., 2004. Stochastic optimal control, international finance and debt. Journal of Banking and Finance 28, 979–996.

Hansen, B. E., 2000. Sample splitting and threshold estimation. Econometrica 68 (3), 575–603.

Kourtellos, A., Stengos, T., Tan, C., 2013. The effect of public debt on growth in multiple regimes. Journal of Macroeconomics 38, Part A, 35–43.

Lane, P. R., Milesi-Ferretti, G. M., 2007. The external wealth of nations mark ii. Journal of International Economics 73 (2), 223–250.

MacLean, L. C., Thorp, E. O., Ziemba, W. T., 2010. Long-term capital growth: the good and bad properties of the kelly and fractional kelly capital growth criteria. Quantitative Finance 10 (7), 681–687.

MacLean, L. C., Thorp, E. O., Ziemba, W. T., 2011. Medium term simulations of the full kelly and fractional kelly investment strategies. In: MacLean, L. C., Thorp, E. O., Ziemba, W. T. (Eds.), The Kelly Capital Growth Investment Criterion. Handbook in Financial Economics vol. 3. World Scientific, pp. 543–561.

Mankiw, N., Elmendorf, D., 1999. Government Debt. North Holland.

Merton, R. C., 1971. Optimum consumption and portfolio rules in a continuous-time model. Journal of Economic Theory 3, 373–413.

Merton, R. C., 1990. Continuous Time Finance. Blackwell, Massachusetts.

Reinhart, C. M., Rogoff, K. S., 2010. Growth in a time of debt. American Economic Review Papers and Proceedings 100 (2), 573–578.

Shreve, S. E., 2004. Stochastic Calculus for Finance II. Springer, New York.

Stein, J. L., 2005. Optimal debt and endogenous growth in models of international finance. Australian Economic Papers 44, 389–413.

Stein, J. L., 2006. Stochastic Optimal Control, International Finance and Debt Crises. Oxford University Press.

Stein, J. L., 2007. United States current account deficits: A stochastic optimal control analysis. Journal of Banking and Finance 31 (5), 1321–1350.

Stein, J. L., 2009. Application of stochastic optimal control to financial market debt crises. CESifo Working Paper No: 2539.

Stein, J. L., 2010. A tale of two debt crises: A stochastic optimal control analysis. Economics: The Open-Access, Open-Assessment E-Journal 4.

Stein, J. L., 2011. The diversity of debt crisis in europe. Cato Journal 31 (2), 199–215.

Stein, J. L., 2012. Stochastic Optimal Control, and U.S. Debt Crises. Springer-Science.

Stein, J. L., Paladino, G., 2001. Country default risk: An empirical assessment. Australian Economic Papers 40 (4).