Stochastic Convergence Rates and Applications of Adaptive Quadrature in Statistical Inference

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Abstract:

Adaptive numerical quadrature is used to approximate (a) the posterior distribution in Bayesian inference and (b) the marginal likelihood in generalized linear mixed models (GLMMs), making it a widely used methodology in modern statistical science. In joint work with Blair Bilodeau and Yanbo Tang, we provide the first convergence rates of the approximation when used in statistical problems, as well as for novel approximations to moments, marginal densities, and quantiles. Where previous known convergence rates relied on assumptions that ruled out all likelihoods, our regularity conditions are classical. We give convergence rates for maximum likelihood estimators in GLMMs fit using adaptive quadrature, and argue that adaptation is necessary to achieve consistent estimation. The rates are conjectured to be tight, and we prove this for the special case of the Laplace approximation. We apply the new theory to give the first formal convergence analysis for a novel extension of the popular "integrated nested Laplace approximation" methodology for approximate Bayesian inference, although we emphasize that work on this topic is still in its infancy.